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Modifying and Reducing Numerical Dissipation in A Two-Dimensional Central-Upwind Scheme

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Abstract. This study presents a modification of the central-upwind Kurganov scheme for approximating the solution of the 2D Euler equation. The prototype, extended from a 1D model, reduces substantially less dissipation than expected. The problem arises from over-restriction of some slope limiters, which keep slopes between interfaces of cells to be Total-Variation-Diminishing. This study reports the defect and presents a re-derived optimal formula. Numerical experiments highlight the significance of this formula, especially in long-time, large-scale simulations.

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1 Introduction

This study simulates the dynamics of ideal gas based on conservation laws for mass, momentum and energy, which are described by the hyperbolic system of Euler equations. Over the last three to four decades, researchers have presented many schemes and improved greatly in this field [9]. The central-upwind (Riemann-problem-solver-free and central Godunov-type projection-evolution) methods [2–8, 11–13] offer impressive advances with key features that bypass solving of the Riemann problem and therefore simplify complex and heavy computations. The central-upwind framework also significantly decreases the numerical dissipation present in the staggered central schemes. They were improved progressively by more precise estimates of the width

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of Riemann fans[†] and a higher order of interpolation with sharper slopes to reconstruct cell distributions. Finally, semi-discrete configurations facilitate multidimensional extensions substantially. As described in [2], this series of algorithms enjoys the advantages of high resolution, simplicity, universality and robustness.

Despite lighter loading, it took at least 320 CPU hours to complete one round of computation for the 2D Rayleigh-Taylor instability problem with 1728×6912 grids. Such time-consuming experiments motivate the use of newest massively parallel computing technique, GPGPU (General purpose computing with graphics processor units). The most valuable feature of a GPU is its large number of scalar processors (SPs) which offer much better computing performance than a CPU (GPU $\approx$ 1000GFLOPS vs. CPU $<$ 10GFLOPS). Here GFLOPS means Giga (one billion) Floating point Operations Per Second. However, the cost is a whole new programming strategy under special structures [15, 16, 19–22].

In this application, we tried several revised configurations and found that the anti-diffusion term (2.5) is the key to improving computing accuracy near discontinuities. Whereas the dissipation reducing is not easily observed over a short period or at a low resolution, the powerful GPU can perform one round of simulation within 13.5 hours. This makes it possible to investigate the ultimate effects of the anti-diffusion mechanism.

In solving 1D Euler equations, the prototype [2] keeps discontinuities as narrow as possible. However, it shows signs of weakness when simulating 2D Euler equations. This is caused by over-operation of some slope limiters in formula (2.3a), whose original purpose is to restrict, under Total-Variation-Diminishing (TVD) conditions, the slopes between interfaces of cells to avoid oscillations. This study analyzes what happens, re-derives a modified formula in Section 3 and finally demonstrates its significance with a series of large-scale numerical simulations in Section 4.

2 Numerical algorithm

The 2D Euler equations can be written as

$$\mathbf{U}_t + f(\mathbf{U})_x + g(\mathbf{U})_y = s(\mathbf{U}), \tag{2.1}$$

where

$$\begin{aligned}
&\mathbf{U}(x,y,t) = (\rho, \rho u, \rho v, E),\\
&f(\mathbf{U}) = f(\rho, \rho u, \rho v, E) = (\rho u, \rho u^2 + P, \rho uv, u(E+P)),\\
&g(\mathbf{U}) = (\rho v, \rho uv, \rho v^2 + P, v(E+P)).
\end{aligned}$$

Here $\rho(x,y,t)$ is the density, $(u(x,y,t), v(x,y,t))$ is the velocity, $E(x,y,t)$ is the total energy and $P(x,y,t)$ is the pressure. The relationship between is

$$E = \frac{P}{\gamma - 1} + \frac{1}{2}P(u^2 + v^2);$$

[†]The fan area is caused by the propagation of discontinuity from the initial state.