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Cosine Radial Basis Function Neural Networks for Solving Fractional Differential Equations

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Abstract. In this paper, we first apply cosine radial basis function neural networks to solve the fractional differential equations with initial value problems or boundary value problems. In the examples, we successfully obtained the numerical solutions for the fractional Riccati equations and fractional Langevin equations. The computer graphics and numerical solutions show that this method is very effective.

AMS subject classifications: 26A33, 60G22

Key words: Fractional Riccati equation, fractional Langevin equation, numerical solution, radial basis function neural network.

1 Introduction

Cosine radial basis function (RBF) neural networks which first introduced by M. Mary and B. Nicolaos are constructed by linear generator functions of a special form [1]. In [1], experimental data indicate that Cosine RBF outperform considerably conventional Cosine RBF neural networks with Gaussian radial basis functions. Cosine RBFs are also strong competitors to existing reformulated RBF models trained by gradient descent and conventional feedforward neural networks with sigmoid hidden units. The more detail about Cosine RBF neural networks, the reader can see [1].

In this paper, by using the method of Cosine RBF neural network, we first obtain the numerical solutions for the fractional Riccati equation of the following form

$$ {}^cD^{\alpha}_{0+}y(x)=f(x)+g(x)y+h(x)y^2, \quad 0<x\leq 1, \quad 0<\alpha\leq 1, \tag{1.1}$$

with initial condition as follows

$$y(0)=c,$$

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where ${}^cD^{\alpha}_{0+}$ is the Caputo fractional derivative of order α, and c is a constant. In case of $\alpha=1$, the fractional equation reduces to the classical Riccati differential equation, and this equation of various forms paly a key role in many fields of robust control, systems identification, dynamic games, calculus of variations, network synthesis, diffusion problems and financial mathematics [2–4]. The value $\alpha=1/2$ is especially popular. This is because in classical fractional calculus, many of the model equations developed used order of $\alpha=1/2$ [5]. Recently, some effective methods for solving fractional Riccati equations have been presented, such as homotopy analysis method (HAM) [6], homotopy perturbation method (HPM) [7], Decomposition method [8]. All these methods tried to obtain the series solutions for the equations. In this paper, we presented a different method for solving fractional Riccati equation, which were constructed from cosine functions with adjustable parameters, and by adjusting the parameters repeatedly to minimize an appropriate error function, we obtain the numerical solutions when the error values are small enough.

We then obtain the numerical solution for the following three-point boundary value problem of fractional Langevin equation with two different fractional orders

$$ {}^cD^{\beta}_{0+}({}^cD^{\alpha}_{0+}+\lambda)y(x)=f(x,y(x)), \quad 0<x\leq 1, \quad 0<\alpha\leq 1, \quad 1<\beta\leq 2, \tag{1.2} $$

satisfying the boundary conditions

$$ y(0)=c_1, \quad y(\eta)=c_2, \quad y(1)=c_3, \quad 0<\eta<1, $$

where ${}^cD^{\alpha}_{0+}$ is the Caputo fractional derivative, $\lambda>0$, c_1, c_2 and c_3 are positive real numbers. The classical Langevin equation was first formulated by Langevin to describe Brownian motion which generated by collisions with the molecules of the fluid, and now it has been used in many areas, such as modelling the evacuation processes [9], photoelectron counting [10], analyzing the stock market [11], etc. However, There are still some phenomena, for example the anomalous diffusion, power-law phenomena, long-range interaction, etc, which can not be accurately described by classical Langevin equation. Therefore, fractional Langevin equations have been presented to model the above behaviors. More detail the reader can see [12–16]. The existence and uniqueness of solution of Langevin equation involving two fractional orders in different intervals ($0<\alpha\leq 1$, $1<\beta\leq 2$) have been studied in [17].

2 Definitions and lemma

Definition 2.1 (see [18])**.** The Riemann-Liouville fractional integral of order $\alpha\in R$, $\alpha>0$ of a function $f(x)\in C_{\mu}$, $\mu\geq -1$ is defined as

$$ I^{\alpha}_{0+}f(x):=\frac{1}{\Gamma(\alpha)}\int_0^x \frac{f(t)dt}{(x-t)^{1-\alpha}}, \quad x>0. $$