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Multi-Symplectic Wavelet Collocation Method for Maxwell's Equations

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Abstract. In this paper, we develop a multi-symplectic wavelet collocation method for three-dimensional (3-D) Maxwell's equations. For the multi-symplectic formulation of the equations, wavelet collocation method based on autocorrelation functions is applied for spatial discretization and appropriate symplectic scheme is employed for time integration. Theoretical analysis shows that the proposed method is multi-symplectic, unconditionally stable and energy-preserving under periodic boundary conditions. The numerical dispersion relation is investigated. Combined with splitting scheme, an explicit splitting symplectic wavelet collocation method is also constructed. Numerical experiments illustrate that the proposed methods are efficient, have high spatial accuracy and can preserve energy conservation laws exactly.

AMS subject classifications: 37M15, 65P10, 65M70, 65T60

Key words: Multi-symplectic, wavelet collocation method, Maxwell's equations, symplectic, conservation laws.

1 Introduction

Maxwell's equations are the most foundational equations in electromagnetism and play an important role in a large number of engineering applications. It is of much significance to develop effective numerical methods to simulate Maxwell's equations with two or three spatial dimensions. Nowadays, a great deal of numerical methods have been used to solve the Maxwell's equations. The finite-difference time-domain (FDTD) method for the one-dimensional Maxwell's equations was first proposed by Yee in [1], which is simple and flexible but is conditionally stable. The FDTD requires large memory and CPU time to obtain accurate solutions for high-dimensional problem. Thus, the FDTD is not a computationally efficient method. Actually, the intensive memory and CPU time requirements mainly come from two aspects: small

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spatial step because of low spatial accuracy and small time step because of stability constraints. For the first pitfall, Krumpholz and Katehi developed a multiresolution time-domain (MRTD) method [2], which has high spatial accuracy but has stringent stability constraints. For the second constraints, some unconditionally stable methods were also constructed. Zheng et al. proposed an ADI-FDTD method for the 3-D Maxwell's equations with an isotropic and lossless medium in [3], but they had't made simulations. Combining splitting method with FDTD scheme, Gao et al. constructed a splitting FDTD method for the 2-D Maxwell's equations and applied the method to solve a scattering problem successfully [4].

On the other hand, during modern numerical simulation procedure, it is significant to construct numerical methods to preserve the intrinsic properties of the original system, such methods are called structure preserving methods containing energy-preserving methods, symplectic methods, multi-symplectic methods, and so on. The ADI-FDTD and splitting FDTD methods may break the energy conservation property of the Maxwell's equations. To overcome the problem, symplectic FDTD scheme [5], which is conditionally stable, energy-conserved splitting FDTD methods [6] and discontinuous Hamiltonian finite element methods [7] were developed. Recently, multi-symplectic algorithms are developed rapidly for multi-symplectic Hamiltonian PDEs due to their long-term behavior and good preservation of local conservation laws [8–19]. However, multi-symplectic schemes are seldom considered for the Maxwell's equations. A new multi-symplectic self-adjoint scheme was developed to simulate the 2-D Maxwell's equations in [20], which is difficult to be applied for 3-D problems. Kong et al. proposed splitting multi-symplectic integrators for Maxwell's equations [22], where the central box scheme was used to discrete the sub-Hamiltonian systems. The 3-D Maxwell's equations were well simulated and good energy-preserving properties were also obtained. However, the convergence rate of the methods is not high. To construct a numerical method which is unconditionally stable and energy-preserving and has high accuracy motivates the current work.

Recently, we have proposed symplectic wavelet collocation method (SWCM) and multi-symplectic wavelet collocation method (MSWCM) for Hamiltonian PDEs in [23, 24]. These methods have the merits of high accuracy, less computations, singularity capturing, invariants preserving properties, and have been applied for solving 2-D Schrödinger equations in [25]. In this paper, we generalize the MSWCM to solve the 3-D Maxwell's equations. The method can obtain expected numerical errors with much less grid points because of high spatial accuracy and takes less computations because of sparse spatial differentiation matrices. We prove that the MSWCM is unconditionally stable and can preserve the two energy conservation laws of the Maxwell's equations exactly under periodic boundary conditions. The dispersion relation for the MSWCM is also investigated. In addition, the properties of the wavelet spectral matrix are analyzed in detail. Then an explicit splitting symplectic wavelet collocation method (ES-SWCM) is also constructed and compared with the MSWCM. Moreover, numerical simulations for the 2-D and 3-D Maxwell's equations are taken and a deep numerical investigation showing the tradeoff between computational effort and error