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Mathematical Development and Verification of a Finite Volume Model for Morphodynamic Flow Applications

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Abstract. The accuracy and efficiency of a class of finite volume methods are investigated for numerical solution of morphodynamic problems in one space dimension. The governing equations consist of two components, namely a hydraulic part described by the shallow water equations and a sediment part described by the Exner equation. Based on different formulations of the morphodynamic equations, we propose a family of three finite volume methods. The numerical fluxes are reconstructed using a modified Roe's scheme that incorporates, in its reconstruction, the sign of the Jacobian matrix in the morphodynamic system. A well-balanced discretization is used for the treatment of the source terms. The method is well-balanced, non-oscillatory and suitable for both slow and rapid interactions between hydraulic flow and sediment transport. The obtained results for several morphodynamic problems are considered to be representative, and might be helpful for a fair rating of finite volume solution schemes, particularly in long time computations.

AMS subject classifications: 65M08, 76B15, 76M12

Key words: Morphodynamic model, shallow water equations, sediment transport, finite volume method, well-balanced discretization.

1 Introduction

The main concern of morphodynamics is to determine the evolution of bed levels for hydrodynamic systems such as rivers, estuaries, bays and other nearshore regions

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where water flows interact with the bed geometry. Example of applications include among others, beach profile changes due to severe wave climates, seabed response to dredging procedures or imposed structures, and harbour siltation. The ability to design numerical methods able to predict the morphodynamic evolution of the coastal seabed has a clear mathematical and engineering relevances. In practice, morphodynamic problems involve coupling between a hydrodynamic model, which provides a description of the flow field leading to a specification of local sediment transport rates, and an equation for bed level change which expresses the conservative balance of sediment volume and its continual redistribution with time. In the current study, the hydrodynamic model is described by the shallow water equations and the sediment transport is modelled by the Exner equation. The coupled models form a hyperbolic system of conservation laws with a source term.

Nowadays, much effort has been devoted to develop numerical schemes for morphodynamic models able to resolve all hydrodynamic and morphodynamic scales. Special attention has been given to the treatment of the source term and the bed-load flux. It is well known that shallow water equations on nonflat topography have steady-state solutions in which the flux gradients are nonzero but exactly balanced by the source terms. This well-balanced concept is also known by conservation property (C-property), compare [4, 20] among others. The well-established Roe's scheme [16] has been modified in [9] for the sediment transport problems. However, for practical applications, this method may become computationally demanding due to its treatment of the source terms. Numerical methods based on Euler-WENO techniques have also been applied to sediment transport equations in [13]. Authors in [6] extended the ENO and WENO schemes to sediment transport equations, whereas the CWENO method has been applied to sediment transport problems in [5]. Unfortunately, most ENO, WENO and CWENO methods that solves real morphodynamic models correctly are still very computationally expensive. On the other hand, numerical methods using the relaxation approximation have also been applied to sediment transport equations in [8]. The relaxation schemes employ general higher order reconstruction for spatial discretization and higher order implicit-explicit schemes or TVD Runge-Kutta schemes for time integration of the relaxing systems. It is well known that TVD schemes have their order of accuracy reduced to first order in the presence of shocks due to the effects of limiters.

The object of this study is to devise a numerical approach able to accurately approximate solution to morphodynamic problems. Our aim is to develop a family of finite volume methods based on a non-homogeneous Riemann solver for solving morphodynamic models. The methods have been investigated in [18, 19] for numerical solution of conservation laws with source terms and are adapted in the current work for the numerical solution of morphodynamic problems. The proposed finite volume scheme belongs to the class of methods that employ only physical fluxes and averaged states in their formulations. To control the local diffusion in the scheme and also to preserve monotonicity, a parameter is introduced based on the sign matrix of the flux Jacobian. The main characteristics of such a finite volume scheme are on