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On the Generation of Instability Tollmien-Schlichting Waves by Free-Stream Turbulence

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Abstract. The beginning of the transition from the laminar to a turbulent flow is usually the generation of instability Tollmien-Schlichting (T-S) waves in the boundary layer. Previously, most numerical and experimental researches focused on generating instability T-S waves through the external disturbances such as acoustic waves and vortical disturbances interacting with wall roughness or at the leading-edge of flat-plate, whereas only a few paid attention to the excitation of the T-S waves directly by free-stream turbulence (FST). In this study, the generating mechanism of the temporal mode T-S waves under free-stream turbulence is investigated by using direct numerical simulation (DNS) and fast Fourier transform. Wave packets superposed by a group of stability, neutral and instability T-S waves are discovered in the boundary layer. In addition, the relation between the amplitude of the imposed free-stream turbulence and the amplitude of the excited T-S wave is also obtained.

AMS subject classifications: 76F65, 76F40, 76D33

Key words: Free-stream turbulence, Tollmien-Schlichting wave, boundary layer.

1 Introduction

Transition from laminar to turbulent flow is one of the key issues in fluid mechanics. The generation of instability T-S waves plays an important role in the laminar-turbulent transition. The first experiment on the generation of T-S waves was carried out by Shubauer and Skramsted [1] with a vibrating ribbon. And in a moderate frequency range, the unsteady suction alone is capable to excite T-S waves, which was first analysed by Gaster [2] on the basis of the Orr-Sommerfeld (O-S) equation. Under the free-stream disturbances, Goldstein [3] showed that T-S waves can be generated near the leading-edge of flat-plate, where the boundary layer is thin and growing rapidly. One more feasible approach to excite T-S waves was demonstrated by Ruban [4], Goldstein [5], Duck et al. [6] and Wu [7]

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through an interaction of free-stream disturbances with wall roughness [8–10]. The most relevant experimental studies on this kind of generation of T-S waves are the one by Wiegel and Wlezien [11] for acoustic disturbances, and the one by Dietz [12] for vortical disturbances. In spite of this, only a few researchers tried to excite instability T-S wave directly by free-stream turbulence. Luo and Zhou [13] studied to generate T-S waves by two harmonic components of the turbulent velocity spectrum though nonlinear interaction. By using a triple-deck formalism, Wu [14] showed that suitable convecting gusts can interact with sound waves in the free stream to produce T-S waves. In this paper, the generating mechanism of T-S waves directly by free-stream turbulence [15] is studied by direct numerical simulation and fast Fourier transformation, and some meaningful results are obtained.

2 Fundamental equations and numerical methods

2.1 Fundamental equations

The dimensionless Navier-Stokes equations are written as

$$\begin{cases} \nabla\cdot\boldsymbol{V}=0, \\ \dfrac{\partial\boldsymbol{V}}{\partial t}+(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}=-\nabla p+\dfrac{1}{Re}\nabla^2\boldsymbol{V}. \end{cases} \tag{2.1}$$

Here, $\boldsymbol{V}=\boldsymbol{U}+\boldsymbol{V}'$, and $\boldsymbol{U}$ is the Blasius solution; $\boldsymbol{V}'=\{u,v\}^T$ is the perturbation velocity; p is the pressure; the Reynolds number is defined as $Re=(U_\infty\delta^*)/v$; $\delta^*$is the boundary-layer displacement thickness; U_∞ is the free-stream velocity; and v is the kinematic viscosity.

2.2 Numerical methods

High-order finite difference schemes are introduced to discretize the fundamental equations. For time integration, a modified fourth-order Runge-Kutta scheme is employed [16]; while a spectral method is used for x-direction discretization and non-uniform compact schemes are used for y-direction discretization [17].

2.3 Free-stream turbulence model

According to [15], the free-stream turbulence model is represented as

$$\begin{pmatrix} u_\infty(x,y,t) \\ v_\infty(x,y,t) \end{pmatrix}=\epsilon\sum_{m=-M}^{M}\sum_{j=-J}^{J}\begin{pmatrix} \hat{u}_\infty(m,j) \\ \hat{v}_\infty(m,j) \end{pmatrix}\cdot\exp[I\cdot(m\kappa_1\cdot x+j\kappa_2\cdot y-m\kappa_1\cdot t)], \tag{2.2}$$

where $I=\sqrt{-1}$; ϵ denotes the amplitude; κ_1 and κ_2 are the x-direction and y-direction fundamental wave numbers respectively; M and J are the maximum mode numbers.