Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **8**, No. 2, pp. 213-235

DOI: 10.4208/aamm.2013.m362
April 2016

High-Order Finite Volume Methods for Aerosol Dynamic Equations

Ming Cui[1,2,*], Yanxin Su[2] and Dong Liang[3,2]

[1] *College of Applied Sciences, Beijing University of Technology, Beijing 100124, China*
[2] *School of Mathematics, Shandong University, Jinan 250100, China*
[3] *Department of Mathematics and Statistics, York University, 4700 Keele Street, Toronto, Ontario, M3J 1P3, Canada*

Received 29 September 2013; Accepted (in revised version) 29 January 2015

Abstract. Aerosol modeling is very important to study the behavior of aerosol dynamics in atmospheric environment. In this paper we consider numerical methods for the nonlinear aerosol dynamic equations on time and particle size. The finite volume element methods based on the linear interpolation and Hermite interpolation are provided to approximate the aerosol dynamic equation where the condensation and removal processes are considered. Numerical examples are provided to show the efficiency of these numerical methods.

AMS subject classifications: 65M10, 65M15, 65N10, 65N15

Key words: Aerosol dynamic equation, finite volume method, condensation, removal.

1 Introduction

In recent years, the acid rain, Antarctic ozone hole, the sand storms of Northern China and many heavy industry cities "gray haze", which are relevant to atmospheric aerosols, have aroused widespread concern in society. Aerosol particles have a major effect on climate change and human health. Hence, the research on atmospheric aerosols is attracting more attention, see [7,10,11]. In the study of aerosols, the particles are classified as nuclei mode, accumulation mode and coarse mode according to the grain diameter.

Nuclei mode ($r < 0.05\mu$m, average radius is about 0.005μm): These particles are formed by the nucleation of gases or small liquid particles, such as S, SO_2 which are emitted by factories, and sulfate particles by oxidation in atmosphere. The particle in nuclei mode mostly comes from industry pollution and has highly concentration. Nuclei mode is the

*Corresponding author.
Email: mingcui@bjut.edu.cn (M. Cui), suyanxin163@163.com (Y. X. Su), dliang@mathstat.yorku.ca (D. Liang)

most unstable mode, and usually transformed to accumulation mode by nucleation and coagulation, which is one of the mainly source of particles in accumulation mode.

Accumulation mode ($0.05\mu\text{m} < r < 1.0\mu\text{m}$, average radius is about 0.1μm): The particles in accumulation mode have weak brown motion, a lower coagulation rate than nuclei mode, and lower dry deposition rate than coarse mode, and can be removed by wet deposition as condensation nucleus of clouds. Accumulation mode is the most stable mode in the three mode. The particles in this mode are the aerosol particles that are harmful to people's health, as they usually have harmful substance, and can be inhaled by lung.

Coarse mode ($r > 1.0\mu$m, average radius is about 5μm): Aerosol particles are mainly formed by some natural and artificial factors of atmosphere transport, human industry and agriculture, and pollution of human life. For example, the concentration of aerosol particles will increase rapidly under sandstorm weather environment. But residence time of particles in coarse mode is usually very short, several days or even several hours, and are mainly removed by dry and wet deposition. As the aggravation of environment pollution, the diameter range of aerosol particles in coarse mode is getting bigger.

Usually, we use the distribution function $n(v,t)dv$, which represents the number of particles per unit volume, at time t, with particle volume between v and $v+dv$ to describe the aerosol system mathematically. In fact, the aerosol particle evolution is governed by the different physical-chemical processes.

We introduce the aerosol dynamic equations when the condensation growth process, the coagulation process and removal of the particles are taken into account. In this paper we only deal with such cases, which condensation growth rate and removal of the particles are taken into account in separate. Hence, the aerosol dynamic equations considered in this paper have the following form:

$$\begin{aligned}&\frac{\partial n(v,t)}{\partial t}+\underbrace{\frac{\partial[G(v)n(v,t)]}{\partial v}}_{Condensation}+\underbrace{R(v)n(v,t)}_{Removal}\\&=\underbrace{\frac{1}{2}\int_{V_{\min}}^{v-V_{\min}}\beta(v-u,u)n(v-u,t)n(u,t)du-n(v,t)\int_{V_{\min}}^{V_{\max}}\beta(v,u)n(u,t)du}_{Coagulation},\end{aligned} \tag{1.1}$$

with the boundary and initial conditions:

$$n(V_{\min},t)=0, \qquad t\in[0,T], \tag{1.2a}$$

$$n(v,0)=n^0(v), \qquad v\in[V_{\min},V_{\max}]. \tag{1.2b}$$

In practice, one assumes that the aerosol particle distribution has a nonzero minimal volume $V_{\min}$ and a finite maximal volume $V_{\max}$. Thus, the aerosol dynamic equations are built on a finite volume interval $\Omega=[V_{\min},V_{\max}]$.

Next, we introduce different process of the aerosol dynamic equation.