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Stability for Leipholz's Type of Laminated Box Columns with Nonsymmetric Lay-Ups on Elastic Foundation

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Abstract. The stability behavior of the Leipholz's type of laminated box columns with nonsymmetric lay-ups resting on elastic foundation is investigated using the finite element method. Based on the kinematic assumptions consistent with the Vlasov beam theory, a formal engineering approach of the mechanics of the laminated box columns with symmetric and nonsymmetric lay-ups is presented. The extended Hamilton's principle is employed to obtain the elastic stiffness and mass matrices, the Rayleigh damping and elastic foundation matrices, the geometric stiffness matrix due to distributed axial force, and the load correction stiffness matrix accounting for the uniformly distributed nonconservative forces. The evaluation procedures for the critical values of divergence and flutter loads with/without internal and external damping effects are briefly presented. Numerical examples are carried out to validate the present theory with respect to the previously published results. Especially, the influences of the fiber angle change and damping on the divergence and flutter loads of the laminated box columns are parametrically investigated.

AMS subject classifications: 65F15

Key words: Leipholz column, laminate, stability, divergence, flutter, finite element analysis.

1 Introduction

The nonconservative forces are the applied forces whose direction changes according to the deformed shape during the course of deformation. Such forces are nonconservative, if the virtual work which they do cannot be represented as the variation of a potential. The fluid-structure interaction is a typical example of distributed follower forces. Païdoussis [1] extensively studied the vibrational behavior of pipes conveying fluid. Examples of concentrated follower forces include the cam rolling on given profile,

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the rotating machinery at the end of a beam, and the solid-fuel rocket mounting. A detailed discussion of this subject and a comprehensive list of references can be found in the book by Leipholz [2] and these nonconservative systems have been analyzed using the various research approaches (e.g., the Lagrangian approach and the assumed mode method [3–6], the finite difference method [2,7–10], the transfer matrix approach [11–14], the finite element method [15–19] and other forms of discretization methods [20,21]).

It is well known that the destabilizing effect caused by the small internal damping has been one of the interesting and important topics in nonconservative stability problems. Because there is no physical system in which damping is not involved in one form or another, research on the stability of columns subjected to nonconservative forces will be very meaningful. As shown in works of many authors [22–28], the realistic modeling of any structure must include the damping effect. Ziegler [22] found that the critical force for small damping can be lower than that evaluated without damping by considering the viscoelastic double pendulum model. Kounadis and Simitses [23] showed more information about this phenomenon for the double pendulum, and the physical explanation of the destabilizing effect of damping was given by Semler et al. [24] on the basis of two-degrees-of-freedom articulated system. Krätzig et al. [25] showed that the critical load decreases about 15% more in the damped system, with dissipation ratios of 3% with respect to the first two eigen-frequencies, than in the undamped one for the Beck's column.

On the other hand, considerable research efforts have been made to investigate the divergence or flutter behavior of nonconservative systems resting on elastic foundation. Smith and Herrmann [29] first showed that the critical flutter load for the cantilever beam is independent of the Winkler foundation modulus for a Bernoulli-Euler beam. Sundararajan [30] also stated that the critical flutter load does not decrease due to the introduction of the Winkler type foundation provided that the modulus distribution of the foundation is geometrically similar to the mass distribution of the beam. Hauger and Vetter [31] observed that the weakening of the foundation can improve the stability of the column, whereas a strengthening of it may decrease the critical load. The generalization of the Smith-Herrmann problem by considering the attachment of the elastic foundation to the part of the column was presented by Elishakoff and Wang [32]. Elishakoff and Jacoby [33] investigated the influence of various types of elastic foundation on the buckling and flutter loads of Ziegler's model structure. Lee and Yang [34] and Lee et al. [35] investigated the influences of the Winkler elastic foundation modulus on the critical load of the uniform and non-uniform Timoshenko beams, respectively, subjected to a concentrated follower force. Recently Kim et al. [36] studied the effects of the subtangentiality, damping, and Winkler foundation stiffness on the stability behavior of the Leipholz's type of columns using the finite element formulation. The existing literatures related to the nonconservative stability of columns on elastic foundation reveal that most of studies so far made have been related to columns made with isotropic material.

In the present work, the stability problems of the Leipholz's type of laminated box columns on elastic foundation are studied. The classical Leipholz column with isotropic