Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **7**, No. 2, pp. 145-157

DOI: 10.4208/aamm.2013.m193
April 2015

The a Posteriori Error Estimates for Chebyshev-Galerkin Spectral Methods in One Dimension

Jianwei Zhou*

Department of Mathematics, Linyi University, Shandong 276005, China

Received 17 April 2013; Accepted (in revised version) 24 February 2014

Abstract. In this paper, the Chebyshev-Galerkin spectral approximations are employed to investigate Poisson equations and the fourth order equations in one dimension. Meanwhile, p-version finite element methods with Chebyshev polynomials are utilized to solve Poisson equations. The efficient and reliable *a posteriori* error estimators are given for different models. Furthermore, the *a priori* error estimators are derived independently. Some numerical experiments are performed to verify the theoretical analysis for the *a posteriori* error indicators and *a priori* error estimations.

AMS subject classifications: 65N30, 65N35, 65M70

Key words: Chebyshev-Galerkin spectral approximation, Chebyshev polynomial, *a posteriori* error indicator, p-version finite element method.

1 Introduction

Either Galerkin spectral methods or p-version finite element methods provide higher accurate approximations with relatively small number of unknowns (see, for example, [5, 8, 20, 21]). In order to get a numerical solution with acceptable accuracy, one may enhance the degrees of polynomial basis if the *a posteriori* error indicators are larger than given criterions (see [2, 7]). However, *a posteriori* error estimations for the p-version FEM and spectral Galerkin methods have been much less developed. In the past decades, there are some papers on this topic in literatures (see, for instance, [4, 13, 14, 17, 19] and the references therein). Recently, Guo summarized some theoretical results of *a posteriori* error estimations for high order finite element methods, Zhou and Yang employed Legendre polynomials to construct an improved *a posteriori* error estimator for Galerkin spectral method in one dimension and analysed its reliability and efficiency, more details please refer to [14, 24].

*Corresponding author.
Email: jwzhou@yahoo.com (J. W. Zhou)

There have been extensive works using finite element methods to analyse the fourth order equations (see [6, 10, 18] and the references cited therein). In many engineering applications, some mathematical models have been described by the first bi-harmonic equations. However, there are less works on *a posteriori* error estimations of Chebyshev-Galerkin spectral methods governed by the first bi-harmonic equations.

This work investigates the *a posteriori* error indicators of spectral Galerkin methods with Chebyshev polynomials for two kinds of model problems in one dimension. The analyses show that the *a posteriori* error estimators can be reformulated with a simple form, which can be easily used in some applications.

The outline of this paper is as follows. In Section 2, we state the model problems in one dimension and recall some well-known results of Chebyshev polynomials and the *a priori* error estimates. In Section 3, we construct the Chebyshev-Galerkin spectral approximation scheme and derive the *a posteriori* error indicator, meanwhile, we prove its efficient and reliable properties. In Section 4, we investigate the p-version finite element methods for the model problem, and study the corresponding *a posteriori* error estimations. In Section 5, we deduce the *a posteriori* error indicator for the fourth order equations in one dimension. Finally, various numerical examples are presented in Section 6 to exhibit the accuracy and efficiency of our results.

2 Poisson equations and Chebyshev-Galerkin spectral approximations

We give some basic notations which will be used in the sequel. Let $\Omega=(-1,1)$ with the boundary set $\partial\Omega=\{-1,1\}$, $\omega(x)$ be a positive weight function on Ω. We shall use the weighted Sobolev spaces $H^m_\omega(\Omega)$ $(m=0,\pm1,\cdots)$ whose norms are denoted by $\|\cdot\|_{m,\Omega}$. In particular, the norm and inner product of $L^2_\omega(\Omega)$ are denoted by $\|\cdot\|_\omega$ and $(\cdot,\cdot)_\omega$, respectively. Meanwhile, we use the standard notations of Sobolev spaces (see [1]). Let $T_k(x)$ be the k-th degree Chebyshev polynomial. Denoting by $\mathcal{S}_N$ the space of polynomials of degree $\le N$, we set

$$\mathcal{S}_N=\text{span}\{T_0(x),T_1(x),\cdots,T_N(x)\},\quad V_N=\{v\in\mathcal{S}_N : v|_{\partial\Omega}=0\},$$

where

$$T_{j+1}=2xT_j-T_{j-1},\quad j=1,2,\cdots,$$

with $T_0=1$, $T_1=x$, $T_2=2x^2-1$.

Let the scalar product in weighted space $L^2_\omega(\Omega)$ as

$$(v,w)_\omega=\int_\Omega vw\omega,$$

where $\omega(x)=1/\sqrt{1-x^2}$.