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Lattice BGK Simulation of Multipolar Vortex Formation

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Abstract. Analytical and numerical studies have shown that multipolar vortices can emerge in two-dimensional flow due to azimuthal normal mode perturbations of shielded vortices. It has been found that mode 2 and 3 perturbations can lead to the formation of stable tripoles and quadrapoles, respectively, while higher order modes result in more complex unstable compound vortices. We have used the lattice Boltzmann method to simulate the effect of azimuthal perturbations on shielded vortices at moderate Reynolds numbers. We have found that azimuthal normal mode perturbations result in the formation of multipoles, which decay due to viscous dissipation. We could also observe that the outcome of such simulations is very sensitive to the displacement of perturbations above wavenumber-3 excitations, in spite of the significant viscosity we used.

AMS subject classifications: 65M10, 76D17, 78M99

Key words: Lattice Boltzmann method, shielded vortex, stability.

1 Introduction

Coherent structures can spontaneously emerge in two-dimensional turbulent flows and the understanding of their formation from small-scale noise might give some insight into turbulent transport. The simplest and most common form of such coherent structures is the monopolar vortex, which contains a single recirculating zone. It is sometimes surrounded by an oppositely-signed vorticity ring, which together with the core form a shielded vortex. Azimuthal normal mode perturbations of shielded vortices can lead to the development of exotic compound coherent structures, so called multipoles [1–4]. For example, stable tripole can emerge from a shielded vortex due

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to the growth and saturation of an azimuthal normal mode with wavenumber-2. The tripole consists of a vortex core and two opposite-sign satellite vortices. Wavenumber-3 can result in the formation of a stable quadrapole with one core vortex bound to three satellites. Strong higher order azimuthal perturbations can lead to the formation of more complex compound structures, which were found to be unstable to infinitesimally small perturbations. Laboratory and numerical experiments are in good agreement with the above observations.

We have used the lattice Boltzmann method (LBM) to simulate the effect of azimuthal perturbations on shielded vortices. The objective of this paper is twofold. The first is to demonstrate that LBM is able to model vortex instabilities. The second is to show that high order multipoles can be formed by adding strong high order azimuthal perturbations to shielded vortices at moderate Reynolds numbers, although these structures are very sensitive to the form and displacement of the initial disturbances.

2 Lattice Boltzmann method

For the simulations presented in this paper we used the LBM with BGK (Bhatnagar-Gross-Krook) collision operator. This method proved to be adequate for many specific fluid dynamics problems involving vortex dynamics (see e.g., [5–7]). The method is based on the solution of the lattice Boltzmann equation (see [8] for further details)

$$f_i\big(\boldsymbol{x}+\boldsymbol{c}_{i\alpha}\delta, t+\delta\big) - f_i(\boldsymbol{x},t) = -\frac{1}{\tau}\big[f_i(\boldsymbol{x},t) - f_i^{eq}(\boldsymbol{x},t)\big], \tag{2.1}$$

where f_i is the one particle velocity distribution function, $c_{i\alpha}$ is the discrete particle velocity, δ is the timestep, i is the index of the lattice links and the local equilibrium distribution function is given as follows

$$f_i^{eq} = \rho w_i\Big(1 + \frac{u_\alpha c_{i\alpha}}{c_s^2} + \frac{u_\alpha u_\beta}{2c_s^4}Q_{i\alpha\beta}\Big), \tag{2.2}$$

where

$$Q_{i\alpha\beta} = c_{i\alpha}c_{i\beta} - c_s^2\delta_{\alpha\beta},$$

w_i is the lattice weight and c_s is the speed of sound. For the simulations presented in this paper, we used a D2Q9 model [9]. The macroscopic quantities are obtained from the distribution functions by taking their moments

$$\rho = \sum_i f_i, \quad \rho u_\alpha = \sum_i c_{i\alpha} f_i. \tag{2.3}$$

The pressure and the kinematic viscosity are given by

$$p = \rho c_s^2, \quad \nu = c_s^2\big(\tau - \frac{1}{2}\big)\delta. \tag{2.4}$$

Recently, we have used the same model to study the interactions between shielded vortices [10].