Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **2**, No. 4, pp. 399-429

DOI: 10.4208/aamm.09-m0917
August 2010

Vortex Capturing Using PNS-WENO Schemes in Uniform and Non Uniform Mesh Formulations

Demian De Feo[1,*], Ning Qin[1] and Trevor J. Birch[2]

[1] *Sheffield University Aerodynamics and Thermofluids Group, Sheffiled, UK*

[2] *Defence Science and Technology Laboratory, Farnborough, UK*

Received 2 February 2009; Accepted (in revised version) 2 February 2010

Available online 28 May 2010

Abstract. High order approximations of the vortical flowfield and resulting aerodynamic coefficients of complex supersonic vortical flows, are computed using the Implicit Parabolized Navier-Stokes solver (IMPNS). Third and fifth order Weighted Essentially Non-oscillating (WENO) schemes for evenly spaced and for stretched structured meshes are employed for the approximate Riemann solution of the inviscid cross flow fluxes. An approximate Riemann solution is obtained using the Osher and Solomon solver and the one-equation Spalart-Allmaras turbulence model is modified for an improved strain-vorticity approximation. Results indicate that even on much coarser meshes the 5$^{\text{th}}$ order PNS-WENO-Spalart-Allmaras approach may achieve results that are superior to previously published full Navier-Stokes solutions that employ a two-equation RANS model but the additional computational demand of schemes for non-uniform grids, may not be justifiable for smoothly varying meshes. The proposed PNS-WENO scheme combination provides a novel approach that is fast, accurate and robust, and that can substantially reduce numerical dissipation and improve the resolution of the vortical structures.

AMS subject classifications: 65M70, 76M20

Key words: PNS solutions, WENO method, vortical flows, supersonic flows.

1 Introduction

Three-dimensional supersonic flows are often characterised by strong viscous/inviscid interactions which often require a prohibitive number of grid points for computational analysis. The computational cost of an accurate Navier-Stokes solution can often be eluded through suitable assumptions applicable to the supersonic problem under consideration. The Parabolized Navier-Stokes equations (PNS) are an approximate form, which through suitable assumptions reduces the complex nature of the

*Corresponding author.
URL: `http://www.shef.ac.uk/fluids/staff/nq.html`
Email: d.defeo@shef.ac.uk (D. M. de Feo), n.qin@sheffield.ac.uk (N. Qin), tjbirch@dstl.gov.uk (T. J. Birch)

Navier-Stokes (NS) equations to a set of mixed parabolic-hyperbolic equations solvable through a space marching approach. This technique of solution ultimately provides considerable savings in computational cost, allowing more resources to be allocated to finer grids for a better resolution of the flowfield.

As constant increases in computing power advance us towards more accurate and detailed numerical flow solutions, more accurate mathematical models substantially increase accuracy without the need for grid alterations. Considerable improvements have been made since the piecewise constant approximation of the original first-order Godunov scheme and current higher order methods offer large benefits at a very affordable computational cost. High resolution schemes are most often applied on the full NS equations and from the authors' research it appears that there is no documented evidence of orders higher than three ever being applied to the PNS equations. Vortical flows are significantly degraded down stream and in the absence of significant mesh refinement around the vortex structure, the dissipation introduced by most numerical algorithms is readily manifested through a non-physical loss of vorticity.

The implicit, multizone, space-marching solver, IMPNS [1–3], was originally developed for the aerodynamic prediction of supersonic viscous flowfields. By employing 3rd (WENO3) and 5th (WENO5) order weighted non-oscillating schemes for equally spaced meshes as well as much more complex 3rd (WENO3-SG) and 5th order (WENO5-SG) formulations for stretched structured grids, the present investigation attempts to minimize the loss in vorticity introduced through grid and discretization errors. This results in a highly efficient and robust solver whose accuracy is on par with more detailed Navier-Stokes solvers but whose savings in terms of memory and computational time of solution are significant. The present investigation also proposes a modification to the production constant in the Spalart Allmaras model which is introduced following the strain-vorticity formulation originally put forward by Dacles-Mariani [4]. Results are obtained for a turbulent tangent ogive-cylinder and found to be superior to well documented results using a much more complete full Navier-Stokes approach with more complex turbulence models on much finer grids. Following the validation, the flowfield over a more intricate geometry possessing four delta type fins placed in an "X" configuration is investigated with the same schemes.

1.1 Reviewing the use of PNS equations and high resolution schemes

The Parabolized Navier Stokes Equations have been applied successfully since the late 1960's but the mixed hyperbolic-parabolic equation set still requires that certain prescribed conditions are met [5]. The conditions require that the inviscid flow in the region outside the boundary layer remain supersonic throughout and that there is no streamwise flow reversal. A further constraint stems from the streamwise pressure gradient which is neglected in many older formulations but may be kept allowing some upstream influence through the boundary layer provided one of several techniques is applied to avoid the departure solutions that would otherwise unfold.

One of the first uses of the PNS equation was by Rudman and Rubin in 1968 [6],