

Ma's Variation of Parameters Method for Fisher's Equations

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Abstract. In this paper, we apply Ma's variation of parameters method (VPM) for solving Fisher's equations. The suggested algorithm proved to be very efficient and finds the solution without any discretization, linearization, perturbation or restrictive assumptions. Numerical results reveal the complete reliability of the proposed VPM.

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Key words: Variation of parameters method, variational iteration method, nonlinear problems, Fisher's equation, nonlinear diffusion equation, error estimates.

1 Introduction

Nonlinear partial differential equations arise in almost all areas of the natural and engineering sciences [1,2,19–21,27,28]. A particular class of equations is those modeling nonlinear reaction and diffusion phenomena [19–21,27,28]. The one-dimensional Fisher equation [19–21,27,28] provides an example for which the diffusion is linear and the reaction term is quadratic in the dependent variable. However, many cases occur in which the diffusion term is either nonlinear or the diffusion coefficients are functions of the dependent variable, see [19–21,27,28] and the references therein. The general properties of the solutions to such equations often depends on the given initial or boundary values selected, consequently a variety of possible behaviors may exist, including wave-like shock solutions [19–21,27,28]. Several numerical and analytical methods including Laurent series, tanh, finite difference and Adomian's decomposition have been developed for solving Fisher's equation, see [19–21,27,28] and the references therein. Most of these techniques have the inbuilt deficiencies. Ma et al. [3–5] used variation of parameters for solving involved non-homogeneous partial

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differential equations and obtained solution formulas helpful in constructing the existing solutions coupled with a number of other new solutions including rational solutions, solitons, positions, negatons, breathers, complexions and interaction solutions of the KdV equations. It is worth mentioning that Ma et al. [3–18] also developed and introduced some other very reliable and revolutionary new techniques also for solving physical problems of nonlinear sciences. The basic inspiration of this paper is the extension of Ma's variation of parameters method [3–5, 22–26] for solving Fisher's equations. It is to be highlighted that such equations arise very frequently in nonlinear sciences and mathematical physics, see [19–21, 27, 28] and the references therein. The VPM is applied without any discretization, perturbation, transformation or restrictive assumptions and is free from round off errors. We apply the proposed VPM for all the nonlinear terms in the problem without discretizing either by finite difference or spline techniques at the nodes, involves laborious calculations coupled with a strong possibility of the ill-conditioned resultant equations which is a too complicated to solve. Moreover, unlike the method of separation of variables that requires initial and boundary conditions, the VPM provides the solution by using the initial conditions only. It is observed that proposed VPM is also easier to implement as compare to the traditional decomposition method since it is independent of the complexities arising in calculating the so-called Adomian's polynomials. We have also applied variational iteration method for solving Fisher's equations and got results which are in full agreement with the results obtained by Ma's variation of parameters method.

2 Variation of parameters method (VPM)

Consider the following second-order partial differential equation

$$y_n = f(t, x, y, z, y_x, y_y, y_z, y_{xx}, y_{yy}, y_{zz}), \quad (2.1)$$

where t such that $(-\infty < t < \infty)$ is time, and f is linear or non linear function of $y, y_x, y_y, y_z, y_{xx}, y_{yy}, y_{zz}$. The homogeneous solution of (2.1) is

$$y(t, x, y, z) = A + Bt,$$

where A and B are functions of x, y, z and t . Using Variation of parameters method, we have following system of equations

$$\begin{aligned} \frac{\partial A}{\partial t} + \frac{\partial B}{\partial t} &= 0, \\ \frac{\partial B}{\partial t} &= f, \end{aligned}$$

and hence

$$\begin{aligned} A(x, y, z, t) &= D(x, y, z) - \int_0^t s f ds, \\ B(x, y, z, t) &= C(x, y, z) - \int_0^t f ds, \end{aligned}$$