Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **2**, No. 3, pp. 259-285

DOI: 10.4208/aamm.09-m09105
June 2010

A Free Streaming Contact Preserving Scheme for the M_1 Model

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Received 04 December 2009; Accepted (in revised version) 25 February 2010

Available online 25 March 2010

Abstract. The present work concerns the numerical approximation of the M_1 model for radiative transfer. The main purpose is to introduce an accurate finite volume method according to the nonlinear system of conservation laws that governs this model. We propose to derive an HLLC method which preserves the stationary contact waves. To supplement this essential property, the method is proved to be robust and to preserve the physical admissible states. Next, a relevant asymptotic preserving correction is proposed in order to obtain a method which is able to deal with all the physical regimes. The relevance of the numerical procedure is exhibited thanks to numerical simulations of physical interest.

AMS subject classifications: 65M06, 85A25.

Key words: Radiative transfer equation, M_1 model, finite volume method, Riemann solver, HLLC scheme, asymptotic preserving scheme.

1 Introduction

The radiative transfer is involved in many applications where its relevant numerical simulation turns out to be essential. However, in several cases where it is coupled with other physics such as hypersonic atmospheric reentry, solving the full radiative transfer equation has a numerical cost beyond the range of the actual computational

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ressources and alternative models must be considered. In recent years, several models have been introduced and the present work is devoted to one of them; namely the M_1 model introduced by Dubroca-Feugeas [12].

The M_1 model is known to satisfy several fundamental physical properties (the list is given below). The purpose of this paper is to derive a numerical method which is able to preserve all of these physical properties. Let us emphasize that the numerical experiments of interest involve all the physical regimes and therefore it is essential to have a numerical scheme that can handle all of them.

The system of equations governing the M_1 model comes from the first two moments of the radiative transfer equation (see Dubroca-Feugeas [12] for further details). The considered model reads as follows

$$\partial_t E + \nabla \cdot \mathbf{F} = c\sigma\big(aT^4 - E\big), \tag{1.1}$$

$$\partial_t \mathbf{F} + c^2 \nabla \cdot \mathbf{P} = -c\sigma\,\mathbf{F}, \tag{1.2}$$

$$\partial_t(\rho\, C_v T) = -c\sigma\big(aT^4 - E\big). \tag{1.3}$$

Here, E denotes the radiative energy and $\mathbf{F} \in \mathbb{R}^2$ the radiative flux vector. The positive constant a is prescribed by physics, while c and σ respectively denote the speed of the light and the opacity. It is to note that the opacity, which will be considered to be constant here for the sake of simplicity, is in general given by non-linear functions of T, E and $\mathbf{F}$ (see [26]). Concerning the radiative pressure $\mathbf{P}$, it is given by

$$\mathbf{P} = \frac{1}{2}\Big(\big(1-\chi(f)\big)\mathbf{I} + \big(3\chi(f)-1\big)\frac{\mathbf{F}\otimes\mathbf{F}}{\|\mathbf{F}\|^2}\Big)E, \tag{1.4}$$

$$\text{with} \qquad \chi(f) = \frac{3+4f^2}{5+2\sqrt{4-3f^2}}, \tag{1.5}$$

where we have introduced the normalized flux vector $\mathbf{f} = \mathbf{F}/cE$, and we have set $f = \|\mathbf{f}\|$.

Let us emphasize that the radiative equations (1.1) and (1.2), issued from the first two moments of the radiative transfer equation, are coupled to the material temperature T governed by Eq. (1.3). We have set ρ the material specific density and C_v the specific heat capacity.

For the sake of simplicity in the notations, we note $U{=}(E,\mathbf{F}){\in}\mathbb{R}^3$ the radiative state vector in the following admissible space

$$\mathcal{A} = \big\{(E,\mathbf{F})^T \in \mathbb{R}^3;\ \ E \geq 0,\ \ f \leq 1\big\}.$$

In the following, $W{=}(E,\mathbf{F},T)^T$ denotes the state vector defined in the admissible space

$$\Omega = \big\{(E,\mathbf{F},T)^T \in \mathbb{R}^4;\ \ (E,\mathbf{F}) \in \mathcal{A},\ \ T \geq 0\big\}.$$

There are two main regimes of interest governed by the parameter σ. The first one associated with $\sigma = 0$ coincides with the free streaming regime given by the hyperbolic