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A Numerical Study of Two-Fluid Models with Pressure and Velocity Relaxation

Svend Tollak Munkejord[1,*]

[1] *SINTEF Energy Research, Sem Sælands veg 11, NO-7465 Trondheim, Norway*

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Abstract. This paper presents a study of pressure and velocity relaxation in two-phase flow calculations. Several of the present observations have been made elsewhere, and the purpose of the paper is to strengthen these observations and draw some conclusions. It is numerically demonstrated how a single-pressure two-fluid model is recovered when applying instantaneous pressure relaxation to a two-pressure two-fluid model. Further, instantaneous velocity relaxation yields a drift-flux model. It is also shown that the pressure relaxation has the disadvantage of inducing a large amount of numerical smearing. The comparisons have been conducted by using analogous numerical schemes, and a multi-stage centred (MUSTA) scheme for non-conservative two-fluid models has been applied to and tested on the two-pressure two-fluid model. As for other, previously tested two-phase flow models, the MUSTA schemes have been found to be robust, accurate and non-oscillatory. However, compared to their Roe reference schemes, they consistently have a lower computational efficiency for problems involving mass transport.

AMS subject classifications: 76T10, 76M12, 65M12, 35L65.

Key words: Two-phase flow, two-fluid model, MUSTA scheme, pressure relaxation, velocity relaxation.

1 Introduction

The modelling of dynamic two-phase flows has a large range of industrial applications, including the transport of oil and gas, energy processes, and safety analyses of nuclear power plants. This kind of modelling is challenging in several ways. First, the Navier–Stokes equations are averaged, see [12]. This brings forward unknown terms for which it is necessary to find models. Unfortunately, the "basic" two-fluid model, in which as many closure terms as possible have been set to equal to zero, has

*Corresponding author.
URL: `http://www.pvv.org/~stm/research/`
Email: stm@pvv.org (S. T. Munkejord)

complex eigenvalues [33]. Further, in its hyperbolic region, the two-fluid model has several waves whose velocity may vary greatly. This, as well as the appearance of non-conservative terms, makes it challenging to construct robust and accurate numerical methods [5, 32, 45].

The two-pressure two-fluid model has an eigenstructure which lends itself much more easily to analysis than that of the "basic" single-pressure two-fluid model. Furthermore, the two-pressure two-fluid model is hyperbolic everywhere, except at the sonic points [35]. However, for a large class of two-phase flow problems of interest, the phasic pressures are so strongly coupled that a pressure-relaxation procedure is required.

Saurel and Abgrall [36] discussed a two-fluid model augmented by a volume-fraction advection equation and so yielding a two-pressure model. It can be thought of as an extension of the Baer and Nunziato [3] model. The two-pressures-with-instantaneous-pressure-relaxation method has been investigated by several researchers [2, 20, 22, 24]. Still, however, there is a need to clarify the potential advantages of this approach, as opposed to using a more "direct" flow model and then solving it using a suitable numerical method for non-conservative balance laws.

The multi-stage centred (MUSTA) scheme [39,41,42] is aimed at coming close to the accuracy of upwind schemes while retaining the simplicity of centred schemes. It does not require any information of the eigenstructure of the model, except for an estimate of the maximum eigenvalue for the Courant-Friedrichs-Lewy (CFL) criterion. Instead, the Riemann problem at the cell interface is approximated numerically by employing a first-order centred scheme on a local grid. The MUSTA scheme has been tested on the Euler equations [39, 44], as well as on a drift-flux two-phase flow model [27] and on the shallow-water equations [17]. Munkejord et al. [28] derived a MUSTA scheme for the two-fluid model with or without an energy equation by using the framework of formally path-consistent schemes of Castro et al. [32] and Parés [6].

The contribution of this paper is to clarify and strengthen previous observations by several authors, that in some cases have not been explicitly stated. First, the MUSTA scheme is applied to and tested on another equation system – a two-pressure two-fluid model. The scheme is found to be robust and accurate, but not efficient. The efficiency penalty is contrary to the hope of Toro [41] of presenting a non-costly scheme.

Next, a *direct comparison* between computations using a single-pressure and those using a two-pressure two-fluid model can be performed due to the use of the MUSTA scheme in each case. Different propositions have been set forth in the literature regarding the practical value of the pressure-relaxation approach, and the present direct comparison is thought to contribute to more certain conclusions. Here it is clearly seen that the two-pressure two-fluid model with instantaneous pressure relaxation converges to the single-pressure two-fluid model, and it should be noted that this includes any instabilities due to complex eigenvalues in the single-pressure two-fluid model. This observation is in agreement with the remarks of Karni et al. [20] and Hérard [19]. A further point to note is that the pressure relaxation is prone to cause significant numerical smearing. The present study therefore confirms the findings of