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# Similarity Solution for the Synchronous Grouting of Shield Tunnel Under the Vertical Non-Axisymmetric Displacement Boundary Condition

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**Abstract.** Similarity solution is investigated for the synchronous grouting of shield tunnel under the vertical non-axisymmetric displacement boundary condition in the paper. The synchronous grouting process of shield tunnel was simplified as the cylindrical expansion problem, which was based on the mechanism between the slurry and stratum of the synchronous grouting. The stress harmonic function on the horizontal and vertical ground surfaces is improved. Based on the virtual image technique, stress function solutions and Boussinesq's solution, elastic solution under the vertical non-axisymmetric displacement boundary condition on the vertical surface was proposed for synchronous grouting problems of shield tunnel. In addition, the maximum grouting pressure was also obtained to control the vertical displacement of horizontal ground surface. The validity of the proposed approach was proved by the numerical method. It can be known from the parameter analysis that larger vertical displacement of the horizontal ground surface was induced by smaller tunnel depth, smaller tunnel excavation radius, shorter limb distance, larger expansion pressure and smaller elastic modulus of soils.

**AMS subject classifications**: 74G15, 74A10, 74Bxx

**Key words**: Vertical non-axisymmetric displacement boundary, stresses and displacements, virtual image technique, cylindrical cavity expansion source, image source, stress harmonic function, shield tunnel, synchronous grouting, maximum grouting pressure.

## 1 Introduction

Construction method of shield tunnel was apprehensively applied to the city highway, underground subway, drainage, power and communication facilities due to the fatly and

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safety construction and the small influences on the surrounding environment. Because of the tunnel excavation technique, the ground movement caused by the shield tunnel excavation can't be completely eliminated no matter how the shield tunnel construction technique is improved. When a ground displacement take place in the construction process of shield tunnel, surface depositing, structures leaning or dumping will be appear, especially in the non-axisymmetric displacement boundary condition on the vertical surface. However, synchronous grouting technique can improve the stability of the surrounding rock and control the ground surface deformation in the shield tunnel construction process.

For the prediction method of the ground settlement caused by shield construction, many literatures and results have been published with the empirical formula, theoretical analytical method, model test, neural network, stochastic medium theory and numerical calculation, etc. For example, Peck [2] proposed an empirical formula for predicting the ground settlement during the tunnel opening based on the monitoring results. Wood [3] pointed out that the ground loss is the main effective factors of the tunnel excavating in soft soil mass. A cumulative probability curve formula was proposed by Attewell and Woodman [5] to predict the vertical ground settlement at the axil of tunnel in soil mass. Sagaseta [6] proposed the analytic solution of three-dimensional surface deformation for the undrained soil deformation due to the ground loss in elastic semi-infinite space. Rowe and Lee [8, 9] pointed out that the gap parameter would have significant effects on the vertical displacement and the magnitude of ground loss during the tunnel opening in the soft soil. Verruijt and Booker [10] proposed the prediction approach of the ground settlements due to deformation of a tunnel in an elastic half plane. Based on the Peck formula and a large number of local experiments, Loganathan and Poulos [11] proposed an analytical prediction technique for tunneling-induced ground movement in clays. Swoboda and Abu-Krisha [12] presented a new field to analyze three-dimensional (3-D) coupled linear flow for Tunnel Boring Machine (TBM) tunnelling in saturated porous medium. Loganathan et al. [13] carried out three centrifuge model tests to assess tunneling-induced ground deformations in clays and their effects on adjacent pile foundations. The specified ground loss values were achieved by reducing the diameter of the model tunnel and designed to simulate the two-dimensional tunnel-induced ground movements. Berg Van der et al. [14] summed up the transverse and longitudinal ground settlement law by the Heathrow fast tunnel deformation monitoring. Li et al. [15] obtained the elastic solution of the spherical cavity expansion under the inclined non-axisymmetric displacement boundary condition, by using coordinate transformation. Zou and Li [17], Zou and He [18], Zou et al. [19], Zou and Su [20] developed the prediction approaches of stress and displacement incorporating the effects of seepage force, hydraulic-coupling, and out-of-planes stress. Zou and Zuo [21] investigated approximate solution of the cylindrical cavity expansion under the inclined non-axisymmetric displacement boundary condition.

Therefore, it can be known that the above researches of the ground surface settlement mainly focused on the empirical method and the numerical method. Moreover,