Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **1**, No. 5, pp. 664-682

DOI: 10.4208/aamm.09-m0912
October 2009

The Ghost Cell Method and its Applications for Inviscid Compressible Flow on Adaptive Tree Cartesian Grids

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Received 05 January 2009; Accepted (in revised version) 05 June 2009
Available online 30 July 2009

Abstract. In this paper, an immersed boundary algorithm is developed by combining the ghost cell method with adaptive tree Cartesian grid method. Furthermore, the proposed method is successfully used to evaluate various inviscid compressible flow with immersed boundary. The extension to three dimensional cases is also achieved. Numerical examples demonstrate the proposed method is effective.

AMS subject classifications: 65M06, 65M50, 76J20

Key words: Ghost cell method, Cartesian grid, adaptive tree method, inviscid compressible flow.

1 Introduction

This paper focuses on the Ghost Cell method (GCM) and its applications for inviscid compressible flow on adaptive tree Cartesian grids. As we all know, a continuous obstacle of computational fluid dynamics (CFD) for configurations with complex geometry is the problem of mesh generation. Although a variety of grid generation techniques are now available, the generation of a suitable grid for a complicated, multi-element geometry is still a tedious, difficult and time-consuming task.

At present, the spatial discretization methods mainly have three approaches [1,2] for dealing with complicated geometry: unstructured grids, body-fitted curvilinear grids, and Cartesian grids. Unstructured grids mainly use triangles in two dimensional flow, tetrahedrons or prisms in three dimension. The advantages lie in the facility of mesh generation for complicated geometry. But the generation is not toilless,

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and still hard to get a good quality grid. Also the memory requirements and computational time are in general high. The main advantage of structured grids follows from the property that the indices i, j, k represent a linear address space (computational space), since it directly corresponds to how the flow variables are stored in the computer memory. Furthermore, more importantly in CFD applications, it gives more accurate results due to the discretisation methods used in most flow solvers. But there are also disadvantages. These are the generation of single structured grids for complex geometries, also time-consuming, and it can produce highly skewed grids too. In order to deal with complicated configurations, multiblock structured grids must be used. However, very long times are still required for the grid generation in the case of complex configurations.

A third alternative is the Cartesian grid approach. Conceptually, this approach is quite simple. Solid bodies are cut out of a single static background mesh and their boundaries represented by different types of cut cell, or solid bodies are equipped with ghost cells using the immersed boundary. Most previous work on Cartesian grids for the compressible Euler equations are based on Cartesian finite volume method [3]. But these methodologies may suffer stability problems when an explicit time step is used, cut cells become very small, and degenerate cells will be encountered. Generally, in two dimensions, a degenerate cell is defined as a cut-cell where the irregularly shaped (embedded) boundary (i) intersects the cell at more than two points or (ii) interacts any cell face at more than 1 point [4]. Some technique must be employed to overcome those problems and time step stability restrictions [4, 5]. Jia et al. [4] present a robust and efficient hybrid cut-cell/ghost-cell method to overcome the degenerate cell, and the heat equations are considered. Several authors [2, 6] use a merging technique, where small irregular cut cell is merged together with a neighboring regular grid cell. By using this merging technique, the conservation is automatically maintained. But this method increases the amount of geometry processing. Other methods include Berger et al. [7, 8] use rotated boxes (h-box method) to enhance stability and, Colella and coworkers [9, 10] use flux-redistribution procedures. Furthermore, embedded or immersed boundary ghost cell methods may be also a good choice, and Cartesian grid finite difference schemes for CFD problems have proven to be quite efficient.

Recently, Sjögreen and Petersson [3] develop an embedded boundary finite difference technique for solving the compressible two- or three-dimensional Euler equations in complex geometries on a Cartesian grid, and slope limiters are used on the embedded boundary to avoid non-physical oscillations near shock waves. Dadone and Grossman [11, 12] provide a novel finite difference ghost cell method on a Cartesian grid, which considers the effect of curvature, and enforces symmetry conditions for entropy and total enthalpy along a normal to the body surface. The results on Cartesian grids indicate that the ghost cell method of [11,12] is remarkably convergent in grid and presents dramatic advantages with respect to the widely used first- and second-order pressure extrapolation techniques on body-fitted polar grids. In above mentioned papers of embedded or immersed boundary ghost cell methods, uniform grid or any grid clustering near the body are used, which must be maintained to the