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Hopf Bifurcations, Drops in the Lid–Driven Square Cavity Flow

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Abstract. The lid-driven square cavity flow is investigated by numerical experiments. It is found that from Re= 5,000 to Re=7,307.75 the solution is stationary, but at Re=7,308 the solution is time periodic. So the critical Reynolds number for the first Hopf bifurcation localizes between Re=7,307.75 and Re=7,308. Time periodical behavior begins smoothly, imperceptibly at the bottom left corner at a tiny tertiary vortex; all other vortices stay still, and then it spreads to the three relevant corners of the square cavity so that all small vortices at all levels move periodically. The primary vortex stays still. At Re=13,393.5 the solution is time periodic; the long-term integration carried out past t_∞=126,562.5 and the fluctuations of the kinetic energy look periodic except slight defects. However at Re=13,393.75 the solution is not time periodic anymore: losing unambiguously, abruptly time periodicity, it becomes chaotic. So the critical Reynolds number for the second Hopf bifurcation localizes between Re=13,393.5 and Re=13,393.75. At high Reynolds numbers Re=20,000 until Re=30,000 the solution becomes chaotic. The long-term integration is carried out past the long time t_∞=150,000, expecting the time asymptotic regime of the flow has been reached. The distinctive feature of the flow is then the appearance of drops: tiny portions of fluid produced by splitting of a secondary vortex, becoming loose and then fading away or being absorbed by another secondary vortex promptly. At Re=30,000 another phenomenon arises—the abrupt appearance at the bottom left corner of a tiny secondary vortex, not produced by splitting of a secondary vortex.

AMS subject classifications: 76D05, 76F06

Key words: Navier-Stokes equations, Hopf bifurcations, chaos.

1 Introduction

The lid-driven square cavity flow has been investigated numerically. At low Reynolds numbers such as Re=100, 1000, 3,200, and 5,000, the solution is stationary; at mid

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Reynolds numbers such as Re=7,500, 10,000, and 12,500, the solution is time periodic; at high Reynolds numbers such as Re=15,000, 17,500, and 20,000, the solution becomes chaotic. As the Reynolds number increases, three kind of solutions appear: stationary, time periodic, and chaotic. The results reported in [21] reveal that two Hopf bifurcations occur and that the critical Reynolds number for the first Hopf bifurcation localizes between Re=5,000 and Re=7,500 and the critical Reynolds number for the second Hopf bifurcation localizes between Re=12,500 and Re=15,000.

But yet, with precision, when and how changes the flow from stationary to time periodic and then from time periodic to chaotic?

This question which has not been addressed in [21] concerns in the first place this research, deserving much more attention because this is another source of disagreement when solving the lid-driven square cavity flow problem: in [25] the critical Reynolds number for the first Hopf bifurcation localizes between Re=7,500 and 10,000; in [14], approximate to Re=8,000; in [1], between Re=8,017.6 and 8,018.8; in [4,29,31], approximate to Re=7,402, Re=8,031.93, Re=8,000, respectively.

The present research determines two Hopf bifurcations with precision: within an interval of length 0.25 for the Reynolds number.

Indeed, from Re=5,000 to Re=7,307.75 the solution is stationary. But at Re=7,308 the solution is time periodic, not stationary. So the critical Reynolds number for the first Hopf bifurcation localizes between Re=7,307.75 and Re=7,308. Time periodical behavior begins smoothly, imperceptibly at the bottom left corner: at a tiny tertiary vortex—all other vortices stay still, and then it spreads to the three relevant corners of the square cavity—all small vortices at all levels move periodically. The primary vortex stays still. On the same hand, at Re=13,393.5 the solution is time periodic; the long-term integration carried out past t_∞=126,562.5, the fluctuations of the kinetic energy look periodic—except slight defects. But at Re=13,393.75 the solution is not time periodic anymore: losing unambiguously, abruptly time periodicity, it becomes chaotic. So the critical Reynolds number for the second Hopf bifurcation localizes between Re=13,393.5 and Re=13,393.75.

Yet, at high Reynolds numbers, for chaotic solutions, another question arises: when will they reach the time asymptotic regime of the flow, the global attractor [41, p. 104]—and how it looks like? In other words, once the numerical experiment runs for a sufficiently long time to make sure the time asymptotic regime of the flow has been reached, what are the distinctive features of the flow?

This interesting question partially addressed in [21] is the second concern of this research. In [21], it was partially addressed because the larger high Reynolds number considered was Re=20,000 and the long-term integration was carried out past the long time t_∞=25,000; whereas this research adds up three more high Reynolds numbers: Re=22,500, 25,000, 30,000, and the long-term integration is carried out past the long time $t_\infty \gg$25,000.

Indeed, at high Reynolds numbers Re=20,000 until Re=30,000 the solution becomes chaotic. The long-term integration is carried out past the long time t_∞=150,000, expecting the time asymptotic regime of the flow has been reached. The distinctive