Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **1**, No. 4, pp. 510-528

DOI: 10.4208/aamm.09-m0916
August 2009

A Practical Algorithm for Determining the Optimal Pseudo-Boundary in the Method of Fundamental Solutions

A. Karageorghis[1,*]

[1]*Department of Mathematics and Statistics, University of Cyprus, P.O.Box 20537, 1678 Nicosia, Cyprus*

Received 21 January 2009; Accepted (in revised version) 03 April 2009
Available online 18 June 2009

Abstract. One of the main difficulties in the application of the method of fundamental solutions (MFS) is the determination of the position of the pseudo-boundary on which are placed the singularities in terms of which the approximation is expressed. In this work, we propose a simple practical algorithm for determining an estimate of the pseudo-boundary which yields the most accurate MFS approximation when the method is applied to certain boundary value problems. Several numerical examples are provided.

AMS subject classifications: 65N35, 65N38, 65K10
Key words: Method of fundamental solutions, elliptic boundary value problems, function minimization.

1 Introduction

The method of fundamental solutions (MFS) [7, 8, 12] is a relatively new meshless method for the solution of certain boundary value problems. In recent years, it has become increasingly popular because of the ease with which it can be implemented for problems in complicated geometries. The MFS is applicable to problems in which a fundamental solution of the operator of the governing equation is known. The solution is then approximated by linear combinations of fundamental solutions in terms of singularities which are placed on a pseudo-boundary lying outside the domain under consideration. A significant step was achieved in the 90's by Golberg and Chen who extended the applicability of the method to boundary value problems governed by inhomogeneous partial differential equations [11]. Since then the MFS has been applied to a wide variety of physical problems ranging from computer-aided design [31] to

*Corresponding author.
URL: `http://www.mas.ucy.ac.cy/english/teachers/karagheorgis_eng.htm`
Email: andreask@ucy.ac.cy (A. Karageorghis)

electrocardiography [32]. Further, the method has proved to be particularly effective for the solution of free boundary problems [17,27] and inverse problems [2,18,22–24].

One disadvantage of the MFS is that it relies on choosing a pseudo-boundary surrounding the boundary of the domain under consideration on which the singularities are placed. The position of this pseudo-boundary is crucial as the accuracy of the method depends on it. In early applications of the method, the positions of the singularities were taken to be among the unknowns [6, 14, 15, 25]. Collocation on a set of boundary points yielded a nonlinear system of equations which had to solved by means of nonlinear minimization routines. Although effective, this approach is computationally expensive. Despite several further attempts [3,29], the optimal placement of the singularities (i.e., the pseudo-boundary) in the MFS is still an open question. In this work, we propose a simple algorithm for determining an estimate of the optimal pseudo-boundary for certain elliptic boundary value problems.

In Section 2, we present in general terms, the type of problems that we examine. In Section 3, we identify the central problem to be addressed, namely the approximate location of the optimal pseudo-boundary, and subsequently describe the algorithm to be used. Several numerical examples are presented in Section 4 which show the efficacy of the proposed algorithm. In Section 5, the algorithm is applied to a biharmonic problem and shown to yield satisfactory results. Finally, some comments and conclusions are given in Section 6.

2 The method

We consider the boundary value problem

$$\begin{cases} \Delta u = 0, & \text{in} \quad \Omega, \\ u = f, & \text{on} \quad \partial\Omega, \end{cases} \tag{2.1}$$

where $\Omega \subset \mathbb{R}^D$, D=2, 3 is a bounded domain with boundary $\partial\Omega$, and f is a given function.

In the MFS [7, 8, 12], the solution u is approximated by

$$u_N(\boldsymbol{c}, \boldsymbol{Q}; P) = \sum_{n=1}^{N} c_n K_D(P, Q_n), \qquad P \in \overline{\Omega}, \tag{2.2}$$

where $\boldsymbol{c}=[c_1, c_2, \ldots, c_N]^T$, $\boldsymbol{Q}$ is a $N-$vector containing the coordinates of the singularities Q_n, $n = 1, \ldots, N$, which lie outside $\overline{\Omega}$, and $K_D(P, Q)$ is a fundamental solution of the Laplace operator given by

$$K_D(P, Q) = \begin{cases} -\dfrac{1}{2\pi} \log \mid P - Q \mid, & D = 2, \\[2ex] \dfrac{1}{4\pi \mid P - Q \mid}, & D = 3, \end{cases} \tag{2.3}$$