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Study of Secondary Instability of a Streaky Boundary Layer Under Spanwise-Localized Free-Stream Vortical Disturbances

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Abstract. Secondary instability of a streaky boundary layer under spanwise-localized free-stream vortical disturbances (FSVD) is investigated using BiGlobal instability analysis. The instability analysis is performed at different streamwise locations and at different instants in a whole time period and the results in space-time $(x-t)$ plane are taken into consideration, such that unstable modes are found in four unstable zones. According to the comparison of mode growth accumulation, we find that the contribution of secondary instability to bypass transition is much more important than T-S waves and that the center strong streak plays the key role in secondary instability. The sinuous type outer mode at center low-speed streak is the dominant unstable mode and this agrees with experimental observation. The inner mode at center high-speed streak also has high growth rate, but its growth cannot keep accumulating, resulting in its unimportant role in secondary instability. All unstable modes found in this paper appear at Fjortoft inflection points of the basic flow in gradient direction, implying that the modes have the same physical nature, i.e., they are caused by inflectional instability of the shear flow.

AMS subject classifications: 76E09

Key words: Secondary instability, BiGlobal instability analysis, streaky boundary layer, bypass transition, Fjortoft inflection point.

1 Introduction

Bypass transition takes place in a boundary layer at high level of environmental disturbances [1]. One typical bypass transition includes the following process [1,2]: Three-dimensional low-frequency components in FSVD induce streamwise streaks in the boundary layer [3–5]; The streak amplitudes increase quickly such that they become saturated

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soon [6,7]; Secondary instability of the streaky boundary layer leads to growth of high-frequency disturbances [8] and results in turbulent spots; The turbulent spots merge to form a turbulent boundary layer.

For isotropic and axisymmetric FSVD, streaks and bypass transition in boundary layers were studied by experimental [3–5], theoretical [6–8] and numerical [9–11] methods. Secondary instability of streaky boundary layers was investigated by using BiGlobal instability analysis [2,12–14] and two kinds of unstable modes were found, i.e., inner mode and outer mode. The inner mode has low phase velocity and the position of critical layer is low. The eigenfunction appears at the bottom of a high-speed streak. On the other hand, the outer mode has high phase velocity and the position of critical layer is high. The eigenfunction appears at high position in the boundary layer. Furthermore, there are two types of outer mode [12,14], i.e., sinuous type and varicose type. The streamwise velocity component of the complex eigenfunction is antisymmetrical in spanwise direction for the sinuous type, while that is symmetrical for the varicose type. Li et al. [15], Zhang [16] and Li et al. [17] also found outer mode in compressible boundary layers and they distinguish two kinds of outer mode by another method, i.e., Y-mode and Z-mode. The eigenfunction of Y-mode appears at the position where the shear of the basic flow is in wall-normal direction, while that of Z-mode at the position where the shear is in spanwise direction.

Furthermore, there is another kind of FSVD, i.e., spanwise-localized FSVD. In the experiments of Westin et al. [18] and Boiko et al. [19], a pipe was set in the vicinity of the plate nose and a loudspeaker was connected to the pipe with a flexible tube, as shown in Fig. 1. Spanwise-localized FSVD were produced by sending pulse signals to the loudspeaker. In the experiments, they found that the streaks induced in the boundary layer are also spanwise-local [18], as shown in Fig. 2. There is a strong streak at the center and a weak streak at each side of the center one. When the strong streak is high-speed, the weak streaks are low-speed and vice versa. Zhang et al. [2] investigated nonlinear evolution of the streaks by numerical computation of nonlinear unsteady boundary-region (NUBR) equations and the numerical results are coincident with the experimental data. In the computation, reference length is 1mm, reference velocity is 6.6m/s, Reynolds number is $Re=446$, nondimensionalized fundamental frequency is $k_1=0.006$ and time period is 1047. For convenience, they also employed time phase $\phi=k_1 t$ to represent time. After a time period, time phase ϕ increases by 2π. Then Zhang and Luo [20] used the method of Zhang et al. [2] to obtain a streaky boundary layer and did preliminary secondary instability at $x^*=160$mm from the leading edge. In order to find unstable modes, Zhang and Luo [20] increased FSVD intensity. Consequently, the streaks have higher amplitude than the experimental data, while their shapes are still coincident with the experimental data, as shown in Fig. 3. Furthermore, in their secondary instability analysis, four unstable modes were found, including one inner mode and three sinuous type outer modes.

On the basis of the streaks evolution results of Zhang et al. [2] and the secondary instability analysis of Zhang and Luo [20], there are still three problems need to be investigated. Firstly, Zhang and Luo [20] only did secondary instability analysis at one