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An Adaptive Time and Space Discretization Approach for Simulating Unsteady Navier-Stokes Flows

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Abstract. In this paper, we develop a technique of adaptive time stepping and combine it with dynamic mesh adaptation to simulate unsteady Navier-Stokes flows over stationary or moving bodies. The second order Backward Differentiation Formula (BDF2) is employed for time discretization and the adaptation of time step is based on the estimation of temporal error. Via a PID (Proportional Integral Derivative) controller or a classical heuristic controller, the size of time step is determined adaptively by the estimate of temporal error and the specified tolerance. In order to eliminate the lag of the adapted mesh behind the unsteady solution, which is associated with the size of time step, a predictor-corrector scheme is adopted in the dynamic mesh adaptation. The efficiency and reliability of the present adaptive time and space discretization approach are validated by the numerical experiments for two- and three-dimensional flows. In the numerical experiments, the behaviors of different error estimators and step-size controllers have also been compared and discussed.

AMS subject classifications: 93C40, 65M50

Key words: Adaptive time stepping, unsteady flow, dynamic mesh adaptation, temporal error estimation, immersed boundary.

1 Introduction

Although the numerical methods and the capacity of modern computers have been developing continuously, computations of complex flows encountered in science and engineering are still extremely time-consuming. As the flow phenomena may progress arbitrarily in the whole spatial and temporal domain, mesh adaptation and adaptive time

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stepping are becoming more and more indispensable, not only for an accurate solution of the governing equations but also for the reduction of computational cost.

For the simulation of physical phenomena with multiple time scales, the size of the fixed time step must be very small. Adaptive time stepping refers to the technique that decreases (or increases) the size of time step when the phenomenon changes rapidly (or slowly). Recently, the adaptive time stepping is becoming more and more attractive. Using an implicit time discretization scheme with variable time step size, Rosam et al. [1] improved the efficiency of phase-field simulation of binary alloy solidification. Tang et al. [2] employed a technique of adaptive time stepping in the soil seismic liquefaction analysis, the key of which is the mixed error estimation of displacement and pore water pressure. Based on the maximum difference between the numerical solutions at two consecutive time steps, Li et al. [3] presented a simple procedure of adaptive time stepping for the Cahn-Hilliard equation, which has multiple time scales for temporal evolution. In [4], Quaife and Biros proposed an adaptive time step approach for the flow of vesicles suspended in Stokesian fluids and the computational cost of error estimate was reduced by using invariant properties of vesicle flows, i.e., constant area and boundary length in two dimensions. Adaptive time stepping for solving Navier-Stokes equations can also be found in literature. Coupez et al. [5] combined the anisotropic mesh adaptation and adaptive time-stepping in the simulation of incompressible Navier-Stokes flows and the time-step is controlled by the interpolation error. In [6], John and Rang presented a systematic study of time stepping schemes with adaptive step controlled by using embedded methods. Applying to the unsteady Navier-Stokes problems some results concerning a posteriori error estimates and adaptive algorithms known for their sub-problems, Berrone and Marro [7] evaluated the real viability of a combined space and time adaptivity for flow problems in engineering.

The combination of dynamic mesh adaptation and adaptive time-stepping for the simulation of unsteady flow has not yet been investigated thoroughly. Most of the dynamic mesh adaptation methods for time-dependent problems adjust the mesh per n time steps according to the initial solution at the first time step. Within the adaptation period, the features of interest may move outside the refined region and this causes a lag of the adapted mesh behind the unsteady solution. This issue is usually addressed by adjusting the mesh frequently [8]. But, choosing the value of n may become more difficult when the variable time step is employed.

In this work, we combine adaptive time stepping and dynamic mesh adaptation to simulate unsteady flows. The adaptation of time step is based on the estimation of temporal error. The time step size is determined via a PID controller [9–11] or a classical heuristic controller [12]. The predictor-corrector scheme proposed by Zhou and Ai [13] for the dynamic mesh adaptation is adopted to eliminate the lag of the adapted mesh behind the unsteady solution. An immersed boundary method named local Domain-Free Discretization (DFD) [14–17] is employed to solve the Navier-Stokes equations, which can simulate moving-boundary flows on a fixed mesh.

The paper is organized as follow. In Section 2, we briefly describe the governing equa-