Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **11**, No. 2, pp. 255-284

DOI: 10.4208/aamm.OA-2018-0078
April 2019

Multiderivative Combined Dissipative Compact Scheme Satisfying Geometric Conservation Law I: Basic Formulations and Performance Evaluation

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Received 19 March 2018; Accepted (in revised version) 23 December 2018

Abstract. In order to improve the resolution power of the hybrid cell-edge and cell-node dissipative compact schemes (HDCS), a series of multiderivative combined dissipative compact schemes (MDCS) have been proposed in this paper. The design concept of the HDCS has been followed to develop the MDCS satisfying the geometric conservation law (GCL) and possessing inherent dissipation. Evaluations of multiple derivatives are included in the MDCS for the purpose of increasing the scheme resolution. The performance of the MDCS is evaluated by theoretical analysis and numerical tests. The multiple derivatives demonstrate their capability in significantly improving resolution power of the MDCS. A MDCS can achieve much higher resolution power than a HDCS with the same order of accuracy. Based on the solutions of the transition and turbulence decay in three-dimensional Taylor-Green vortex, a fifth-order MDCS with three tri-diagonal operators is recommended. This MDCS has better performance than the fifth-order HDCS in resolving multiple-scales turbulent structures.

AMS subject classifications: 76Fxx, 76Gxx

Key words: Multiderivative combined dissipative compact scheme, multiderivative formulation, dissipative interpolation, geometric conservation law, complex geometry.

1 Introduction

A family of hybrid cell-edge and cell-node dissipative compact schemes (HDCS) [1] were previously proposed. The HDCS has the capability of satisfying the geometric conservation law (GCL) and successful applications on complex grids [2–4]. However, there is a

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disadvantage in this scheme. The accuracy of the proposed boundary and near boundary schemes are fourth-order, while the interior scheme of the recommended HDCS in the [1] with optimized resolution power has seventh-order of accuracy. This mismatching order between the interior and boundary scheme may lead to a computational accuracy loss, since the formal accuracy of the overall scheme is reduced by lower order boundary schemes to one greater than that of the boundary [5]. Although this disadvantage can be overcome by higher-order boundary schemes, it is always found difficult to derive very high-order and robust boundary schemes for pragmatic simulations. It is an alternative in this paper to improve the resolution power of interior schemes. If high resolution power is possessed by a relatively low-order interior scheme, a correspondingly low-order boundary closure can be found for the interior scheme without a computational accuracy loss.

It is well known that the performance of compact difference schemes can benefit from high-order spatial derivatives involved. In order to achieve a higher resolution in a narrower stencil, first and second derivatives are evaluated simultaneously in combined compact difference schemes (CCD) [6–9]. In the CCD, all the derivatives employed on the left-hand side need to be solved together using Hermitian polynomials. It is convenient that one can obtain the solutions of first and second derivatives at the same time. However, it is true that the solving system of the CCD is increasingly complex with more and more high-order spatial derivatives expected to be involved for advanced resolution performance. It is one of the motives for this paper that the solving system can be simplified by moving the high-order derivatives to the right-hand side. In a relatively simple solving system, it is demonstrated in this paper that the high-order derivatives on the right-hand side can be evaluated to greatly improve the resolution performance of compact difference schemes.

There are two types of compact difference schemes. One is central compact difference schemes, another is dissipative compact difference schemes. The central compact difference schemes were systematically analyzed by Lele [10]. In these schemes, the ones formulated from the variables on cell-nodes are called CCSN here. The CCSN has been successfully applied by Visbal et al. [11, 12] for complex simulations with the help of satisfying the GCL. Successful applications of the central compact difference schemes have also been presented in computational aeroacoustic field [13–17]. High-order filters are usually needed by the compact difference schemes on account of suppressing numerical contaminants caused by dispersion errors. Another satisfactory way is to develop a scheme having dissipation [9, 18], namely dissipative scheme.

Compact schemes with dissipation were proposed by Tolstykh [19] through a Murman-type switch [20]. Tolstykh further constructed compact dissipative schemes with arbitrary order of accuracy via linear combinations of third-order operators [21]. Adams and Shariff [22] introduced a free parameter to optimize the resolution power of compact dissipative schemes. It also can be noted that some other compact dissipative schemes have been proposed [23, 24]. For the development of compact difference schemes with inherent mechanism handling unwanted numerical errors, dissipative