Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **11**, No. 1, pp. 108-131

DOI: 10.4208/aamm.OA-2018-0026
February 2019

Numerical Simulation of Non-Linear Schrödinger Equations in Arbitrary Domain by the Localized Method of Approximate Particular Solution

Yongxing Hong[1], Jun Lu[2,3,*], Ji Lin[1] and Wen Chen[1]

[1] *State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, International Center for Simulation Software in Engineering and Sciences, College of Mechanics and Materials, Hohai University, Nanjing 211100, China*
[2] *Nanjing Hydraulic Research Institute, Hujuguan 34 Road, Nanjing 210024, China*
[3] *State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Nanjing 210098, China*

Received 23 January 2018; Accepted (in revised version) 21 September 2018

Abstract. The aim of this paper is to propose a fast meshless numerical scheme for the simulation of non-linear Schrödinger equations. In the proposed scheme, the implicit-Euler scheme is used for the temporal discretization and the localized method of approximate particular solution (LMAPS) is utilized for the spatial discretization. The multiple-scale technique is introduced to obtain the shape parameters of the multi-quadric radial basis function for 2D problems and the Gaussian radial basis function for 3D problems. Six numerical examples are carried out to verify the accuracy and efficiency of the proposed scheme. Compared with well-known techniques, numerical results illustrate that the proposed scheme is of merits being easy-to-program, high accuracy, and rapid convergence even for long-term problems. These results also indicate that the proposed scheme has great potential in large scale problems and real-world applications.

AMS subject classifications: 00A35

Key words: Schrödinger equation, Localized method of approximate particular solution, Shape parameters, Multiple-scale technique.

1 Introduction

In this paper, we consider the following general form of the non-linear Schrödinger equation:

$$\mathtt{i}\frac{\partial u(\boldsymbol{x},t)}{\partial t}+a\Delta u+w(\boldsymbol{x},t)u(\boldsymbol{x},t)+v(\boldsymbol{x},t)|u(\boldsymbol{x},t)|^m u(\boldsymbol{x},t)=0, \tag{1.1}$$

*Corresponding author.
Email: lujunhhu1981@126.com (J. Lu)

under the following initial condition

$$u(\boldsymbol{x},t)=u_0(\boldsymbol{x},t),\quad \boldsymbol{x}\in\boldsymbol{\Omega},\quad t=0, \tag{1.2}$$

and boundary conditions

$$\frac{\partial u(\boldsymbol{x},t)}{\partial \boldsymbol{n}}=f(\boldsymbol{x},t),\quad \boldsymbol{x}\in\Gamma_{\mathrm{N}},\quad t\in[0,T], \tag{1.3a}$$

$$u(\boldsymbol{x},t)=g(\boldsymbol{x},t),\quad \boldsymbol{x}\in\Gamma_d,\quad t\in[0,T], \tag{1.3b}$$

where $\mathrm{i}=\sqrt{-1}$, T is the final time, t denotes the time history, $u(\boldsymbol{x},t)$ is an unknown wave function to be determined, Δ is the Laplace operator, $\omega(\boldsymbol{x},t)$ and $v(\boldsymbol{x},t)$ are known functions and m is a positive real constant, u_0, g, and f are given functions, $\boldsymbol{\Omega}$ denotes the interest domain, Γ_{N} and Γ_d represent boundaries under the Neumann and Dirichlet boundary conditions respectively.

The Schrödinger equations describe various types of physical phenomena in science and engineering. Korepin and Zhang utilized Schrödinger equations to depict wave-corpuscle duality of microscopic particles in quantum mechanics [1, 2]. Onorato and Osborne analyzed the water wave propagation using the Schrödinger equations [3, 4], which can also be used on ocean environmental description and the design of ships and shore structures [5, 6]. The Schrödinger equations have also been used to describe the standing waves [7], electromagnetic fields [8], electro-optic wave propagation [9], as well as the structures of biological materials [10, 11].

In recent years, various numerical algorithms have been proposed to solve the Schrödinger equations. It is known to all that the tradition element-based methods are very powerful and effective tools for solving problems in science and engineering such as the finite difference method, the finite element method, and the boundary element method [12, 13]. Anastassi has proposed the finite difference method in solving the Schrödinger equations and related oscillatory problems [14]. Zhu employed the finite element method for the time-space-fractional Schrödinger equation [15]. To avoid the mesh of the solution domain, the boundary element method reduces the dimension of the problem by one [16]. Zhao has improved the accuracy in solving the fractional Schrödinger equation by the alternating direction implicit scheme [17]. Bhrawy introduced an effective spectral scheme to solve the multi-dimensional space-time variable-order fractional Schrödinger equations [18]. Zhang has proposed the improved complex variable moving least-squares Ritz method and the improved complex variable element-free Galerkin method for Schrödinger equations [19, 20].

In the last two decades, the radial basis functions (RBFs) based methods have attracted great attention and enjoyed considerable success in solving partial difference equations (PDEs) [21–24]. These approaches can be easily extended to solve high dimensional problems due to its spatial dimension independence, and applied to solve high order differential equations due to the smooth characteristic of RBFs [25–28]. It should be noted that there are some drawbacks for the radial basis functions based methods. For some RBFs, it is difficult to obtain the optimal shape parameters, such as the