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Quadrotor Convergence Trajectory Optimization Model Based on Minimum Energy Consumption

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Abstract. Energy consumption is an important challenge to planning path of quadrotor and the energy-calculating of quadrotor flight trajecotry is not yet fully completed. This paper mainly researches on optimizing the convergence trajectory of the quadrotor on energy-saving. Firstly, the equivalence between the lift angular accelerated function and the flight trajectory function is proved. Then, the energy-calculating formula of quadrotor flight trajectory is obtained by using the relationship between lift and voltage. Furthermore, the quadrotor convergence trajectory model is established with the goal of minimum energy consumption. Finally, through the numerical simulation, the optimized convergence trajectories under different boundary conditions are obtained. Moreover, the specific flight position in real time can be got and implemented into the real-time flight trajectory planning.

AMS subject classifications: 65K05, 65C20

Key words: Quadrotor, convergence trajectory, energy, trajectory planning.

1 Introduction

In recent years, quadrotor has gradually become a hotspot in the field of aircraft because of its convenience of take-off and landing, good flexibility and security. At present, the research on the quadrotor is mainly focused on the aspects of flight control, aircraft structure optimization design, path planning, trajectory optimization and so forth. In the design of the quadrotor, it is a key point whether the quadrotor can smoothly achieve an energy-efficient flight and complete the specific task during the process of flight and the track optimization is an important way to solve this problem. Because of the short life of

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the quadrotor's battery, it is very important to reduce the energy consumption. So it is of practical significance to carry out the research on its trajectory optimization.

The optimization of the aircraft trajectory (including the quadrotor) is actually an optimal control problem with state and control constraints. Four types of methods are discussed about optimizing the quadrotor trajectory, the first ones are based on geometric model search, which including probability graph method [1, 2], fast expanding random tree [3, 4], Voronoi graph [5], A* algorithm [6], Dijkstra [7], D*-lite, greedy search etc.. The second ones are based on virtual potential field and the navigation function, which including artificial potential field, harmonic function field. The third ones are based on mathematical optimization, which including rolling optimization control [8,9], nonlinear optimization [10], level set [11]. The fourth ones are based on biological intelligence, which including ant colony algorithm [12], particle swarm algorithm [13], genetic and evolutionary algorithm [14], neutral network etc..

In terms of optimizing the quadrotor trajectory for saving energy, in order to improve the energy efficiency of the aircraft, previous studies have focused on the aspects of the hardware structure optimization such as controller [15–17], rotor [17, 18] and path planning which including geometric model search [12, 14, 19] etc.. Most of the trajectory optimization models, which consider energy-saving do not explicitly obtain an energy-calculating formula [10, 20–23], while the optimization models which propose energy-calculating formula do not take account of the characteristics of quadrotor [24, 25]. Commonly convergence curves include arc curve, Bézier curve and Dubins curve [26–31], but the convergence trajectories of arcs usually do not consider making the curvature equality [30, 31] so that smooth steering can not be guaranteed. Moreover, the convergence trajectories using specific curves such as Bézier curves and Dubins curves [26–29] do not consider the energy-saving, so the least energy consumption can not be guaranteed. Therefore, based on the above work, this paper aims to study the convergence trajectory of the aircraft to overcome the above two limitations.

In this paper, firstly, the dynamic model of the quadrotor is analyzed [10, 32] and the equivalence between the lift angular velocity function and the flight trajectory function is proved under the known flight initial position. Secondly, by using the relationship between the rotor lift and voltage [34], the rotor's power is integrated to get the energy-calculating formula of the quadrotor trajectory. Thirdly, under the constraint of parameters such as lift, voltage and angular velocity, the flight trajectory function optimization model with the goal of consuming the minimum energy is establised. Finally, the trajectory function which satisfies the minimum energy consumption is obtained by the numerical simulation of the problem.

2 The rotor's lift model of the quadrotor's convergence trajectory