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Immersed Finite Element Method for Eigenvalue Problems in Elasticity

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Abstract. We consider the approximation of eigenvalue problems for elasticity equations with interface. This kind of problems can be efficiently discretized by using immersed finite element method (IFEM) based on Crouzeix-Raviart P1-nonconforming element. The stability and the optimal convergence of IFEM for solving eigenvalue problems with interface are proved by adopting spectral analysis methods for the classical eigenvalue problem. Numerical experiments demonstrate our theoretical results.

AMS subject classifications: 65N30, 65N25

Key words: Immersed finite element, elasticity problem, eigenvalue.

1 Introduction

In this paper, we consider the approximation of eigenvalue problems with interface in elasticity. Eigenvalue analysis is an essential basis for many types of engineering analysis. As eigenvalues are closely related with the frequency and shape of structures, computing the eigensolutions is important to interpret the dynamic interaction between the structures. If the frequency of structures is close to the system's natural frequency, mechanical resonance occurs. It may lead to catastrophic failure or damage in constructed structures such as bridges, buildings, and towers [24]. In addition, eigenvalue analysis is applied to stability analysis for many physical problems such as thermoelastic problems [48] and fluid-solid interaction problems [5, 10, 20].

There have been mathematical studies of finite element methods for eigenvalue problems. We begin by pointing out the fundamental references [4, 21, 22, 43] for the analysis of eigenvalue problems. Babuška and Osborn provide the spectral analysis by using the

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properties of compact operators [4, 43]. The spectral approximation together with the case of general operators is presented in [21, 22]. In [46] various computed examples for Laplacian eigenproblems in planar regions are studied and there are references to physical problems where the results are relevant. For a nonconforming approximation of elliptic eigenvalue problems, it is shown that the eigenvalues computed by finite element methods give lower bounds of the exact eigenvalues whose eigenfunctions are singular in non-convex polygon [3]. The guaranteed lower and upper bounds of eigenvalues based on the nonconforming finite element approximation are given in [16]. Moreover, let us focus on eigenvalue problems in elasticity. A posteriori error estimator for linearized elasticity eigenvalue problems is studied in [47]. It is shown that upper and lower estimates for the error of eigenpairs are established in terms of a residual estimate and lower-order terms. In [42], a method for three-dimensional linear elasticity or shell problems is presented to derive computable estimates of the approximation error in eigenvalues. The spectral problem for the linear elasticity equations on curved non-convex domains, as well as with mixed boundary conditions is considered in [27]. Meddahi et al. [40] present an analysis for the eigenvalue problem of linear elasticity by means of a mixed variational formulation. This method weakly imposes the symmetry of the stress tensor and is free from the locking phenomenon.

When the elastic body is occupied by heterogeneous materials, it is known that governing equations contain the discontinuous material parameters along the interface of materials. To simulate such problems by finite element methods, a common strategy is to use fitted meshes along the interface. However, this strategy may require a very fine mesh near the interface. As an alternative approach, some numerical methods using unfitted meshes are proposed. One approach is an extended finite element method (XFEM) introduced by adding enrichment functions to the classical finite element space [23, 41]. Theoretical and computational results for XFEM in elasticity problems can be found in [7–9, 25, 32, 44].

Another method is an immersed finite element method (IFEM) [17,18,30,35,36] which can use any meshes independent of interface geometry. The idea of an IFEM is to construct local basis functions to satisfy the interface conditions without additional degrees of freedoms. For source problems with interface in elasticity, Kwak et al. [29] present a nonconforming IFEM based on the broken Crouzeix-Raviart (CR) element [19]. They prove optimal error estimates and provide numerical results for compressible and nearly incompressible materials. Computational results of IFEM based on the rotated Q_1-nonconforming element are reported in [37] and the related work in this direction can be found in [38]. In addition, the spectral analysis of IFEM for elliptic eigenvalue problems with an interface is given in [31]. Liu et al. [39] introduced a method which bears the same name *immersed finite element* to solve fluid solid interaction problem, but it is different from ours since they use double grids; one for solid another for fluid.

In this work, we analyze the spectral approximation of elasticity interface problems using P_1-nonconforming IFEM and derive the optimal convergence of eigenvalues. Moreover, we provide a series of numerical results of the eigenproblems with various