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# An Efficient High Order Well-Balanced Finite Difference WENO Scheme for the Blood Flow Model

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**Abstract.** The blood flow model admits the steady state, in which the flux gradient is non-zero and is exactly balanced by the source term. In this paper, we present a high order well-balanced finite difference weighted essentially non-oscillatory (WENO) scheme, which exactly preserves the steady state. In order to maintain the well-balanced property, we propose to reformulate the equation and apply a novel source term approximation. Extensive numerical experiments are carried out to verify the performances of the current scheme such as the maintenance of well-balanced property, the ability to capture the perturbations of such steady state and the genuine high order accuracy for smooth solutions.

**AMS subject classifications**: 65L12, 74S20

**Key words**: Blood flow model, finite difference scheme, WENO scheme, high order accuracy, well-balanced property.

## 1 Introduction

In this paper, we are interested in numerical computing the blood flow model in arteries by high order schemes. Numerical simulations with high order accuracy for the blood flow model have wide applications in medical engineering [1,2]. In one spatial dimension, the blood flow model takes the following system of hyperbolic balanced laws [3]:

$$\begin{cases} A_t+Q_x=0, \\ Q_t+\left(\dfrac{Q^2}{A}+\dfrac{K}{3\rho\sqrt{\pi}}A^{\frac{3}{2}}\right)_x=\dfrac{KA}{2\rho\sqrt{\pi}\sqrt{A_0}}(A_0)_x, \end{cases} \tag{1.1}$$

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where $A=\pi R^2$ is the cross-sectional area with $R$ being the radius of the vessel, $Q=Au$ denotes the discharge, $u$ means the flow velocity, $\rho$ stands for the blood density and $K$ represents the arterial stiffness. In addition, $A_0=\pi R_0^2$ is the cross section at rest ($u$=0m/s) with $R_0$ being the radius of the vessel, which may be variable in the case of aneurism, stenosis or taper.

The important property of the model (1.1) is that it admits the steady state, also called mechanical equilibrium, where the flux gradient is exactly balanced by the source term:

$$u=0 \quad \text{and} \quad A=A_0. \tag{1.2}$$

Under the above steady state (1.2), the flux gradient is non-zero and is exactly balanced by the source term. Consequently, it is desirable to maintain the balancing between the flux gradient and the source term at the discrete level. In general, the standard numerical schemes generally fail to satisfy the discrete version of this balance exactly at the steady state, even introduce spurious oscillations, unless the mesh size is extremely refined. However, the mesh refinement procedure is not applicable for practical problems due to the very high computational cost. In 1996, Greenberg et al. [4] originally introduced the well-balanced schemes, which can preserve exactly the steady state solutions up to machine accuracy at the discrete level. Moreover, well-balanced schemes can capture small perturbations well even on relatively coarse meshes [5]. It is important to note that many attempts have been concentrated on the well-balanced schemes for the shallow water equations using different approaches, see among others [6–11] and references therein.

In recent years, there have been many interesting attempts on the well-balanced schemes for the blood flow model. For example, Delestre and Lagrée [12] presented a well-balanced finite volume scheme based on the conservative governing equations [13–15]. Müller et al. [16] constructed a well-balanced high order finite volume scheme for the blood flow in elastic vessels with varying mechanical properties. Recently, Murillo et al. [17] have presented an energy-balanced approximate solver with upwind discretization for the source term. More recently, Wang et al. [18] have presented a well-balanced finite difference WENO scheme based on the splitting algorithm of the source term.

The key objective of this research is to develop an efficient high order well-balanced finite difference WENO scheme based on a reformation of the source term to avoid the splitting of the source term as in [18]. Rigorous numerical analysis as well as extensive numerical experiments all verify the satisfaction of the well-balanced property of the resulting scheme. In order to obtain well-balanced finite difference WENO schemes, we firstly reformulate the source term in an equivalent form, then construct linear finite difference operator coupled with modification of the flux splitting. The above procedures lead to an efficient WENO scheme compared with the WENO scheme in [18]. In addition, this WENO scheme keeps high order accuracy for smooth solutions, and enjoys steep discontinuity transition at the same time.

This paper is organized as follows: we briefly review the key idea of the finite difference WENO schemes in Section 2. In Section 3, we propose a high order well-balanced