Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **10**, No. 1, pp. 1-21

DOI: 10.4208/aamm.OA-2017-0060
February 2018

A Direct Discontinuous Galerkin Method with Interface Correction for the Compressible Navier-Stokes Equations on Unstructured Grids

Jian Cheng*, Huiqiang Yue, Shengjiao Yu and Tiegang Liu

School of Mathematics and Systems Science, Beihang University, Beijing 100091, China

Received 27 February 2017; Accepted (in revised version) 24 April 2017

Abstract. Since the original DDG method has been introduced by Liu et al. [8] in 2009, a variety of DDG type methods have been proposed and further developed. In this paper, we further investigate and develop a new DDG method with interface correction (DDG (IC)) as the discretization of viscous and heat fluxes for the compressible Navier-Stokes equations on unstructured grids. Compared to the original DDG method, the newly developed DDG (IC) method demonstrates its superior in delivering the optimal order of accuracy under demanding situations. Strategies in extension and application of this newly developed DDG (IC) method for solving the compressible Navier-Stokes equations and special treatments designed for handling boundary viscous fluxes are presented and examined in this work. The performance of the new DDG method with interface correction is carefully evaluated and assessed through a number of typical test cases. Numerical experiments show that the new DDG method with interface correction can achieve the optimal order of accuracy on both uniform structured grids and nonuniform unstructured grids, which clearly indicates its potential for further applications of real engineering practices.

AMS subject classifications: 65M60, 76N15

Key words: Direct discontinuous Galerkin method, high-order method, compressible Navier-Stokes equations.

1 Introduction

Discontinuous Galerkin (DG) methods [1–3,16,18], as a typical representative in the community of high order methods, have been widely used in computational fluid dynamics, computational acoustics, and computational magneto-hydrodynamics. The DG methods combine two advantageous features commonly associated with finite element (FE)

*Corresponding author.
Email: chengjian@buaa.edu.cn (J. Cheng)

and finite volume methods (FVM). As in classical FE methods, accuracy is obtained by means of high-order polynomial approximation within an element rather than by wide stencils as in the case of FVM. Similar to FVM, the physics of wave propagation is, however, accounted for by solving the Riemann problems that arise from the discontinuous representation of the solution at element interfaces.

DG methods are indeed a natural choice for solving hyperbolic equations, such as the compressible Euler equations. However, the DG formulation is far less certain and advantageous for diffusion equations such as the compressible Navier-Stokes equations, where viscous and heat fluxes exist and require the evaluation of the solution derivatives at the interfaces. Taking a simple arithmetic mean of the solution derivatives from the left and right is inconsistent, because it does not take into account a possible jump of the solutions. Thus, how to appropriately address this issue becomes one of the core challenges for the practice of DG methods for real applications.

The first attempt at using DGM to solve elliptic and parabolic problems can be tracked back to the late 1970s and early 1980s when an interior penalty (IP) method was independently proposed and studied in [20, 21]. In the IP method, a viscous flux is obtained through the average of the left and right state and then penalizing with a penalty term which includes the jump of the solution at each cell interface. The IP method with different stabilization terms was further analyzed for the two dimensional compressible Navier-Stokes equations by Hartmann et al. [22, 23], which led to a new symmetric IP (SIP) method. Inspired by the great success of the DGM for a first-order system, a natural choice to solve a second-order system is to convert it into a first-order system by introducing additional variables, and then to apply a DG method directly to the first-order system. Based on different choices of numerical flux at the cell interface, there are mainly two kinds of approaches: one is the first Bassi-Rebay (BR1) scheme [18], and the other is the so-called local discontinuous Galerkin (LDG) method [15]. A variation of a LDG method, termed compact DG (CDG) method [17], was later developed by Peraire and Persson to overcome the issue that LDG method is not compact when applied to multidimensional problems. For the same reason, Bassi and Rebay introduced the second Bassi and Rebay scheme (BR2) based on BR1 scheme in order to maintain the compactness and stability for the pure diffusion problems [14]. In practice, the auxiliary variables in both BR2 scheme and CDG method are usually eliminated by introducing the so-called local and global lifting operator. From the finite volume community, van Leer et al. [6, 7] proposed a recovery-based DG (RDG) method for diffusion equations using the recovery principle, that recovers a smooth continuous solution that in the weak sense is indistinguishable from the discontinuous discrete solution. Similarly, Luo et al. [4, 5] developed a reconstructed DG (rDG) method for the compressible Navier-Stokes equations on arbitrary grids, where a smooth continuous solution is reconstructed at each cell interface from the discontinuous discrete solution and the diffusive fluxes are then obtained based on the smooth reconstructed solution. Besides, a kind of hybridizable DG (HDG) discretization was recently introduced for the solution of convection-diffusion equations [24]. For an uniform analysis and comparison among several of the methods