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Size-Dependent Geometrically Nonlinear Free Vibration of First-Order Shear Deformable Piezoelectric-Piezomagnetic Nanobeams Using the Nonlocal Theory

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Abstract. This article investigates the geometrically nonlinear free vibration of piezoelectric-piezomagnetic nanobeams subjected to magneto-electro-thermal loading taking into account size effect using the nonlocal elasticity theory. To this end, the size-dependent nonlinear governing equations of motion and corresponding boundary conditions are derived according to the nonlocal elasticity theory and the first-order shear deformation theory with von Kármán-type of kinematic nonlinearity. The effects of size-dependence, shear deformations, rotary inertia, piezoelectric-piezomagnetic coupling, thermal environment and geometrical nonlinearity are taken into account. The generalized differential quadrature (GDQ) method in conjunction with the numerical Galerkin method, periodic time differential operators and pseudo arc-length continuation method is utilized to compute the nonlinear frequency response of piezoelectric-piezomagnetic nanobeams. The influences of various parameters such as non-dimensional nonlocal parameter, temperature change, initial applied electric voltage, initial applied magnetic potential, length-to-thickness ratio and different boundary conditions on the geometrically nonlinear free vibration characteristics of piezoelectric-piezomagnetic nanobeams are demonstrated by numerical examples. It is illustrated that the hardening spring effect increases with increasing the non-dimensional nonlocal parameter, positive initial applied voltage, negative initial applied magnetic potential, temperature rise and decreases with increasing the negative initial applied voltage, positive initial applied magnetic potential and length-to-thickness ratio.

AMS subject classifications: 35XX, 65XX, 74XX

Key words: Piezoelectric-piezomagnetic nanobeams, geometrically nonlinear free vibration, nonlocal elasticity theory, size effect, magneto-electro-thermal loading.

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1 Introduction

Nanostructured elements such as nanobeams, nanoplates and nanoshells have been widely used as main components in nano- and micro-electro-mechanical systems (NEMS and MEMS). Especially, nano-sized structures made of the piezoelectric-piezomagnetic materials have attracted a great deal of attention in many research interests due to their outstanding inherent magneto-electro-thermo-mechanical coupling effects [1–6]. As the typical component in the NEMS and MEMS, piezoelectric-piezomagnetic nanobeams, nanoplates and nanoshells have a wide range of applications in nano actuators, transducers, resonators and robotics [7–12]. Hence, understanding the linear and nonlinear static and dynamic mechanical behaviors of piezoelectric-piezomagnetic nanostructures is essential for their applications. The investigation of geometrically nonlinear free vibration of piezoelectric-piezomagnetic nanostructures in the thermal environment is a major topic of current interest, which is utilized to fully realize the dynamic characteristic of piezoelectric-piezomagnetic nanostructures in the large amplitude vibrations.

There are some typical experiments such as nano/micro-bend test, nano/micro-torsion test and micro/nano indentation test [13–19] and atomistic simulations [20–22], which have reported the size-dependent mechanical characteristics of small-scale structures. Hence, it is of great signicance to consider the size effect in mechanical characteristics of piezoelectric-piezomagnetic nanostructures. Since the molecular dynamics simulations for large-scale nanostructures are restricted by computational capacities and conducting controlled experiments and operating precision at the nanoscale are difcult, the size-dependent continuum theories including the nonlocal elasticity theory [23], s-train gradient elasticity theory [24], modified couple stress theory [25], modified strain gradient theory [26] and surface stress elasticity theory [27, 28] have been proposed and have been widely utilized to develop the size-dependent beam, plate and shell models for the analysis of size-dependent mechanical characteristics of these small-scale structures [29–37]. Among these theories, the Eringen's nonlocal elasticity theory is commonly utilized to develop the nonlocal continuum models in which the effect of small scale parameter is incorporated.

Recently, several studies have been performed to study the size-dependent static and dynamic behaviors of nanostructures based on the nonlocal elasticity theory. More recently, the nonlocal theory has been extended to investigate the size-dependent mechanical behaviors of the piezoelectric-piezomagnetic nano-scale structures. Ke and his co-authors studied the size-dependent thermoelectric-mechanical free vibration [38], geometrically nonlinear free vibration characteristics [39] and buckling and postbuckling [40] of piezoelectric nanobeams by means of the proposed linear and nonlinear Timoshenko beam models and nonlocal elasticity theory. Moreover, Ke et al. [41, 42] performed a series of studies to investigate the thermo-electro-mechanical vibration of piezoelectric nanoplates using the classical and first-order shear deformable plate theories. Asemi et al. [43] examined the geometrically nonlinear free vibration of piezoelectric nanoelectromechanical resonators. In another work, a nonlocal Love thin shell model