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EFFICIENT AND ACCURATE NUMERICAL METHODS FOR LONG-WAVE SHORT-WAVE INTERACTION EQUATIONS IN THE SEMICLASSICAL LIMIT REGIME*

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Abstract

This paper focuses on performance of several efficient and accurate numerical methods for the long-wave short-wave interaction equations in the semiclassical limit regime. The key features of the proposed methods are based on: (i) the utilization of the first-order or second-order time-splitting method to the nonlinear wave interaction equations; (ii) the application of Fourier pseudo-spectral method or compact finite difference approximation to the linear subproblem and the spatial derivatives; (iii) the adoption of the exact integration of the nonlinear subproblems and the ordinary differential equations in the phase space. The numerical methods under study are efficient, unconditionally stable and higher-order accurate, they are proved to preserve two invariants including the position density in L^1. Numerical results are reported for case studies with different types of initial data, these results verify the conservation laws in the discrete sense, show the dependence of the numerical solution on the time-step, mesh-size and dispersion parameter ε, and demonstrate the behavior of nonlinear dispersive waves in the semi-classical limit regime.

Mathematics subject classification: 65M06, 65M12
Key words: Long-wave short-wave interaction equations, Semiclassical limit, Time-splitting method, Spectral method, Compact finite difference method, Conservative properties.

1. Introduction

In this paper, we aim to construct several efficient and accurate numerical methods to solve the long-wave short-wave interaction (LSI) equations

$$i\varepsilon\partial_t\psi^\varepsilon + \frac{\varepsilon^2}{2}\partial_{xx}\psi^\varepsilon - \left(\alpha(|\psi^\varepsilon|^2 - 1) + \phi^\varepsilon\right)\psi^\varepsilon = 0, \qquad x \in \mathbb{R}, \quad t > 0, \tag{1.1}$$

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$$\partial_t \phi^\varepsilon = -\lambda \partial_x |\psi^\varepsilon|^2, \qquad x \in \mathbb{R}, \quad t > 0, \tag{1.2}$$

where the parameter ε is analogous to the Planck constant in quantum mechanics, the complex-valued function ψ^ε and the real-valued function ϕ^ε present the envelope of the high-frequent short-wave and the amplitude of the long-wave, respectively. The coupling between the long-wave and the short-wave is described by the real parameter λ which introduces the dispersion interaction. The nonlinearity in the coupled system is due to α, $\alpha > 0$ represents the defocusing nonlinearity and $\alpha < 0$ means the focusing nonlinearity. The initial conditions are given as

$$\psi^\varepsilon(x,0) = \psi_0^\varepsilon(x) = A_0(x)\exp(iS_0(x)/\varepsilon), \qquad x \in \mathbb{R}, \quad t > 0, \tag{1.3}$$

$$\phi^\varepsilon(x,0) = \phi_0^\varepsilon(x), \qquad x \in \mathbb{R}, \quad t > 0, \tag{1.4}$$

where A_0 and S_0/ε are the amplitude and phase angle of ψ_0^ε, respectively. The small parameter ε shows the space and time scales introduced in Eqs. (1.1)-(1.2), as well as the typical wave length of oscillations of the initial data. This is precisely the semiclassical limit in the particular case of Schrödinger equation with vanishing Planck's constant (i.e., $\varepsilon \to 0$), which motivates us to consider the LSI system in the semiclassical limit regime, i.e., the system with $0 < \varepsilon \ll 1$.

For $t \geq 0$, the LSI system (1.1)-(1.4) satisfies the following conservative laws [16]:

$$\mathrm{E}_1(t) := \int_{-\infty}^{\infty} |\psi^\varepsilon(x,t)|^2 \, dx \equiv \mathrm{E}_1(0), \tag{1.5}$$

$$\mathrm{E}_2(t) := \varepsilon \int_{-\infty}^{\infty} \mathrm{Im}\,(\partial_x \psi^\varepsilon(x,t)/\psi^\varepsilon(x,t)) \equiv \mathrm{E}_2(0), \tag{1.6}$$

$$\mathrm{E}_3(t) := \int_{-\infty}^{\infty} \phi^\varepsilon(x,t) \equiv \mathrm{E}_3(0), \tag{1.7}$$

$$\mathrm{E}_4(t) := \int_{-\infty}^{\infty} \left[\varepsilon \mathrm{Im}\left(\partial_x \psi^\varepsilon(x,t)\overline{\psi^\varepsilon(x,t)}\right) + \frac{1}{2\lambda}|\psi^\varepsilon(x,t)|^2\right] dx \equiv \mathrm{E}_4(0), \tag{1.8}$$

$$\mathrm{E}_5(t) := \int_{-\infty}^{\infty} \partial_t \phi^\varepsilon(x,t)\,|\psi^\varepsilon(x,t)|^2 \, dx \equiv \mathrm{E}_5(0) = 0, \tag{1.9}$$

$$\begin{aligned}\mathrm{E}_6(t) := \int_{-\infty}^{\infty} \bigg[&\frac{\varepsilon^2}{2}|\partial_x \psi^\varepsilon(x,t)|^2 + \frac{\alpha}{2}\left(|\psi^\varepsilon(x,t)|^2 - 1\right)^2 \\ &+ \phi^\varepsilon(x,t)\,|\psi^\varepsilon(x,t)|^2\bigg] \equiv \mathrm{E}_6(0),\end{aligned} \tag{1.10}$$

where $\mathrm{Im}(f)$ and $\overline{f}$ present taking the imaginary part and conjugate of the function f.

In [14], under the assumptions of long-wave short-wave resonance, Benny proposed several systems of dispersive equations. One of them is given by Eqs. (1.1)-(1.2) which has frequently been used to describe interactions between long-waves and short-waves in various physical settings (see [14,17,20,21,30,32,38]). For example, the Eqs. (1.1)-(1.2) with $\alpha = 0$ was derived in [21] to model the interaction between the long gravity waves and capillary waves on the surface of shallow water, in the case when the group velocity of capillary wave coincides with the velocity of the long-wave. It is pointed out that the physical significance of Eqs. (1.1)-(1.2) is that the dispersion of the short-wave is balanced by the nonlinear interaction of the long-wave with the