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THE FACTORIZATION METHOD FOR A MIXED SCATTERING PROBLEM FROM A BOUNDED OBSTACLE AND AN OPEN ARC*

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Abstract

In this paper, we consider the scattering problem of time-harmonic electromagnetic waves from an infinite cylinder having an open arc Γ and a bounded domain D in $\mathbb{R}^2$ as cross section. We focus on the inverse scattering problem, that is, to reconstruct the shape of Γ and D from the far-field pattern by using the factorization method. Through establishing a mixed reciprocity relation, we prove that the scatters Γ and D can be uniquely determined by the far-field pattern. Furthermore, the mathematical basis is given to explain that the factorization method is feasible to our problem. At the end of this paper, we give some numerical examples to show the efficaciousness of the algorithms.

Mathematics subject classification: 35Q65, 35C15, 78A45
Key words: Factorization method, Inverse scattering problem, Mixed scattering, Time-harmonic electromagnetic wave.

1. Introduction

In this paper, we consider the scattering problem of a time-harmonic electromagnetic waves located outside an infinite cylinder (not perfectly) having an open arc Γ and a bounded domain D in $\mathbb{R}^2$. Some suitable boundary conditions are given on ∂D and Γ, then the scattering problem is modeled by an exterior boundary value problem for Helmholtz equation in the domain $\mathbb{R}^2 \setminus (\bar{D} \cup \bar{\Gamma})$,

$$\begin{cases} \Delta u + k^2 u = 0, & \text{in } \mathbb{R}^2 \backslash (\overline{D} \cup \overline{\Gamma}), \\ \mathfrak{B}_1 u = f, & \text{on } \partial D, \\ \mathfrak{B}_2 u = h, & \text{on } \Gamma, \\ \lim_{r\to\infty} \sqrt{r}(\frac{\partial u}{\partial r} - iku) = 0, & \text{with } r = |x|, \end{cases} \tag{1.1}$$

where k is the wave number and $\mathfrak{B}_i (i = 1, 2)$ denote the boundary conditions on ∂D and Γ. Such problems arise in many areas such as medical imaging, non-destructive testing, geophysical exploration.

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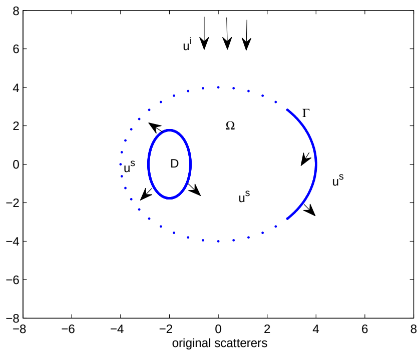
For a sound-soft obstacle, the scattered field u satisfies the Dirichlet boundary condition

$$\mathfrak{B}_1 u := f \quad \text{on } \partial D, \tag{1.2}$$

for an imperfect crack, the scattered field u satisfies the impedance boundary conditions

$$\mathfrak{B}_2 u := \begin{cases} \dfrac{\partial u_+}{\partial \nu} + \lambda_1 u_+ = h_1, & \text{on } \Gamma, \\ \dfrac{\partial u_-}{\partial \nu} - \lambda_2 u_- = h_2, & \text{on } \Gamma. \end{cases} \tag{1.3}$$

where the normal ν to the crack Γ is assumed to exist almost everywhere except a finite number of points, and $\lambda_1, \lambda_2 \in L^\infty(\Gamma)$ are given impedance functions. In this paper, the scattered wave u is radiated by the plane incident wave $u^i = e^{ikx\cdot d}$ ($d \in \mathbb{S}^1$, $\mathbb{S}^1$ denotes the unit circle).



Remark 1.1. (i) $u_\pm(x) = \lim_{h\to 0} u(x \pm h\nu)$ and $\frac{\partial u_\pm}{\partial \nu} = \lim_{h\to 0} \nu \cdot \nabla u(x \pm h\nu)$ for $x \in \Gamma$. In the following discussion, $(\cdot)_\pm$ or $(\cdot)^\pm$ have a similar meaning, that is, they denote the limit approaching the boundary from outside and inside with respect to the corresponding domain. (ii) Assuming the arc Γ can be extended to a piecewise smooth closed curve $\partial\Omega$ enclosing a bounded domain Ω such that the normal vector ν on Γ coincides with the normal vector on $\partial\Omega$, the domain D is completely contained in Ω, i.e., $D \subset \Omega$ and $\partial D \cap \partial\Omega = \emptyset$.

The last condition in (1.1) is the well-known Sommerfeld radiation condition, following the scattered field $u(x, y)$ to have the asymptotic behavior

$$u(x, y) = \frac{e^{ik|x|}}{\sqrt{|x|}} u^\infty(\hat{x}, y) + \mathcal{O}(|x|^{-\frac{3}{2}}) \quad \text{as} \quad |x| \to \infty \tag{1.4}$$

uniformly in all directions $\hat{x} = x/|x|$. The function $u^\infty(\hat{x}, y)$ is called the far-field pattern corresponding to the scattered field $u(x, y)$.

In recent years, the inverse scattering problems for acoustic and electromagnetic waves have been extensively studied (refer to [1, 2, 5–7]), because of the wide applications such as radar, medical imaging, and geophysical explorations, etc. But in the reality, the scatters may be a combination of all kinds of obstacles, such as a bounded fragment and a piece of cable. For convenience, we choose an open arc Γ and a bounded obstacle D represent all kinds of scatters. As we know, the studies are few for this mixed scattering problem. In this paper, we want to what will happen when the scatterer is not a simple crack or not a bounded domain.