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THE DESIGN OF BÉZIER SURFACE THROUGH QUINTIC BÉZIER ASYMPTOTIC QUADRILATERAL*

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Abstract

The asymptotic curve is widely used in astronomy, mechanics and numerical optimization. Moreover, it shows great application potentials in architecture. We focus on the problem how to cover bounded asymptotic curves by a freeform surface. The paper presents the necessary and sufficient conditions for quadrilateral with non-inflection being asymptotic boundary curves of a surface. And then, with given corner data, we model quintic Bézier asymptotic quadrilateral interpolated by a smooth Bézier surface of bi-eleven degree. We handle the available degrees of freedom during the construction to get an optimized result. Some representative surfaces bounded by asymptotic curves with lines or inflections are also discussed by examples. The presented interpolation scheme for the construction of tensor-product Bézier surfaces is compatible with the CAD systems.

Mathematics subject classification: 65D07.
Key words: Asymptotic curves, Bézier surface, Interpolation, Quadrilateral.

1. Introduction

The curve's asymptotic direction is a direction along which the normal curvature is zero. If the tangent vector $\mathbf{e}$ at each point of the curve $\mathbf{r}$ on a surface $\mathbf{R}(u, v)$ is an asymptotic direction of the surface $\mathbf{R}(u, v)$, then the curve is an asymptotic curve of the surface. Asymptotes have been applied in many areas, such as astronomy [1], mechanics [2], numerical optimization [3], architecture [4] and relevant subjects.

Architectural geometry [5] addresses challenges in the realization of complex freeform structures and technical advantages of ruled surfaces. A typical example is hyperbolic paraboloids with negative Gaussian curvatures. Moreover, asymptotic curves can only exist on this kind of negatively curved surface and there are two asymptotic directions at each point of the anticlastic surface-regions. Based on the analysis of these asymptotic curves, this locally saddle-shaped regions can be approximated by ruled surface. Flöry and Pottmann [4] just used asymptotic directions, which were estimated from the given point cloud, to deduce the layout of production sized panels and construct an initial ruled surface by aligning rulings with asymptotic curves.

However, except analyzing the asymptotic curves or interpolating some points as the asymptotic boundary by large ruled surface patches or multiple strips of ruled surfaces, if there exists other general surfaces in practical application?

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In fact, since asymptotic curves are the only type to combine the benefits of straight unrolling and orthogonal nodes along a double-curved grid, they show great application potentials in architectural design. A remarkable realization of this structure is provided by 'Asymptotic Gridshell', which was developed by Eike Schling and Denis Hitrec at TU Munich (see Fig. 1.1 or website *https://www.lt.ar.tum.de/en/research-pavilion/* in detail). They designed developable curved support structures along asymptotic parameter lines and orientated normal to minimal surface [6]. Recent work in [7] generalizes above structure to discretization of asymptotic nets with constant ratio of principle curvatures. The definition of discrete asymptotic net comes from discrete A-net in discrete differential geometry [8]. Also based on this asymptotic gridshell structure, [9] uses a special network to discretize a surface of constant mean curvatures, which shows that when the network becomes an asymptotic one, the surface is a minimal surface. In above freeform structure, beams are asymptotic curves lying on the underlying surface and together with the developable lamelles show good aesthetic qualities. The coverage of a freeform surface is also a main problem in architectural geometry. If we fill in the gridshell by the underlying surface patches, then each patch of the grid is bounded by asymptotic curves of itself. This locally inverse reconstruction problem motives our interest. Then it comes to a basic geometric problem that how to construct a surface possessing the given curves as its asymptotic boundary curves.

Fig. 1.1. 'Asymptotic Gridshell' forms the INSIDE\OUT pavilion. (Picture: Hui Wang)

The study on reconstruction of smooth surfaces from a given boundary curve is a hot topic. Many researchers studied on the construction of surfaces interpolating the special curves, but most of which are geodesics and lines of curvature. Wang et al. [10] constructed parametric surfaces which possess a given spatial curve as a geodesic, and Li et al. constrained a line of curvature as the boundary curve of the surfaces [11]. Kasap et al. [12] and Li et al. [13] extended the parametric surfaces into a generalization of surface family interpolating a geodesic curve and a line of curvature, respectively. Li et al. also designed developable surfaces through Bézier geodesics [14] and a given line of curvature [15]. The number of given curves of these constructed surfaces are at most two. However, Farouki et al. gave the sufficient and necessary conditions of closed geodesic curves [16] and lines of curvature [17] as boundary curves of Coons quadrilateral patches. Based on the conditions in [16], he constructed triangular Coons patches [18] with geodesic boundary curves and four-sided Bézier patches [19] with Bézier curves as geodesic