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Global Existence of Solutions of the Navier-Stokes-Maxwell System in Besov Spaces

Haifeng Bai and Li Li*

Department of Mathematics, Harbin Institute of Technology, Harbin 150001, P.R. China.

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Abstract. The motion of hydro-magnetic fluid can be described by Navier-Stokes-Maxwell system. In this paper, we prove global existence and uniqueness for the solutions of Navier-Stokes-Maxwell system in 3 dimensional space for small data.

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Key words: Navier-Stokes-Maxwell system, Besov space, existence, uniqueness, Littlewood-Play operator.

1 Introduction

Consider the Navier-Stokes-Maxwell (NSM) system in $\mathbb{R}^3$

$$\begin{cases} u_t-\triangle u+u\cdot\nabla u+\nabla p=j\times B, \\ E_t-\nabla\times B=-j, \\ B_t+\nabla\times E=0, \\ \mathrm{div} u=0, \qquad \mathrm{div} B=0, \\ u(x,0)=u_0(x), \quad B(x,0)=B_0(x), \quad E(x,0)=E_0(x), \end{cases} \tag{1.1}$$

where u is the velocity field of the hydro-magnetic fluid, E and B are electrical and magnetic fields respectively, p is the pressure, j is the current density, which is governed by Ohm's law

$$j=\sigma(E+u\times B).$$

The constant σ is resistance, u_0, B_0, E_0 are initial values. Without loss of generality, we can choose the electric resistivity $\sigma=1$ hereinafter. The aim of this paper is to prove the existence of mild solutions of (1.1) in Besov spaces.

*Corresponding author. *Email addresses:* `baihfqing@hit.edu.cn` (H. Bai), `leeeee@hit.edu.cn` (L. Li)

Definition 1.1. We say that (u,E,B) is a *mild solution* of (1.1) if it satisfies the integral equations

$$u(t)=e^{t\triangle}u_0-\int_0^t e^{(t-t')\triangle}\mathcal{P}(\nabla(u\otimes u)(t')-j\times B)\mathrm{d}t',$$
$$\begin{pmatrix}E\\B\end{pmatrix}=e^{tL}\begin{pmatrix}E_0\\B_0\end{pmatrix}-\int_0^t e^{(t-t')L}\begin{pmatrix}v\times B\\0\end{pmatrix}\mathrm{d}t',$$

where $\mathcal{P}:L^2\to\{u\in L^2:\mathrm{div}u=0\}$ is Leray's projection operator

$$\mathcal{P}(u)_j(x)=\frac{1}{(2\pi)^{n/2}}\sum_k\int_{\mathbb{R}^n}\Big(\delta_{jk}-\frac{\xi_j\xi_k}{|\xi|^2}\hat{u}_k(\xi)\Big)e^{i\xi\cdot x}\mathrm{d}\xi,$$

and L is the matrix operator

$$L=\begin{pmatrix}-I & \mathrm{curl}\\ -\mathrm{curl} & 0\end{pmatrix}.$$

The motion of charged viscous fluid can be described by the Navier-Stokes-Maxwell equations, which was derived based on the classical Newtonian dynamics and Maxwell's electromagnetism theory.

In 2010, Masmoudi [16] obtained the global existence of the regular solutions of the Navier-Stokes-Maxwell system in $\mathbb{R}^2$, and proved the exponential growth rate. Duan [8] derived the Navier-Stokes-Maxwell system from the Vlasov-Maxwell-Boltzmann system by using the macro-micro decomposition of Liu, *et al.* [15], he also prove the global existence of solutions for the Cauchy problem of the compressible NSM system and analyzed the large time behavior of the solutoins. In 2015, Germain *et al.* [10] study the well-posedness of mild solutions of the NSM system. Chen and Jügel [6] analyzed the global existence of the solutions for the incompressible Navier-Stokes-Maxwell-Stefan system and proved that the global solution converges to the homogeneous steady state in time. Yang and Wang [21] proved that the regular solution of the compressible NS-M system converges to the regular solution of the incompressible NSM system by using a new energy functional. In 2016, Tan and Tong [19] studied the global existence and large time behavior of the compressible Navier-Stokes-Maxwell system in $\mathbb{R}^3$ with linear damping. Liu and Su [14] obtained global existence near the constant steady states for the non-isentropic Navier-Stokes-Maxwell system. Ibrahim *et al.* [18] established the existence and asymptotic stability for the time periodic small solutions of the incompressible Navier-Stokes-Maxwell system in the whole $\mathbb{R}^3$.

The study on the weak solutions of partial differential equations usually proceed in suitable function spaces, Sobolev spaces for examples. Besov spaces can be viewed as an extension of the classical Sobolev spaces. So the well-posedness of the solutions in Besov spaces has theoretical value for mathematicians, especially in the case when the well-posedness is unknown in classical Sobolev spaces. In 1992, Kobayashi and Muramatu [12] proved the local existence, uniqueness and regularity of the solutions for