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On Compressible Smooth Viscous Fluids in Slowly Expanding Balls

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Abstract. In [17] and [19, 20], the global existence and large time behaviors of smooth compressible fluids (including inviscid gases of Euler equations, viscous gases of Navier-Stokes equations, and rarified gases of Boltzmann equation, respectively) have been established in an infinitely expanding ball with a constant expansion speed. This paper concerns with the viscous fluids in a slowly expanding ball. By involved analysis on the density function and the weighted energy estimates, we show that the fluid in the slowly expanding ball smoothly tends to a vacuum state and there is no appearance of vacuum in any part of the expansive ball. Our present result is a meaningful supplement to the one in [19].

Key Words: Compressible Navier-Stokes equations, slowly expanding ball, weighted energy estimate, global existence.

AMS Subject Classifications: 35L70, 35L65, 35L67, 76N15

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1 Introduction

In this paper, as in [17] and [19, 20], we continue to study the global existence and stability of a smooth compressible viscous flow in a 3-D slowly expanded ball. The slowly expanded ball at time t is described by $S_t=\{x:|x|=\sqrt{x_1^2+x_2^2+x_3^2}\le R(t)\}$, where $R(t)\in C^4[0,\infty)$ satisfies $R(0)=1$, $R'(0)=0$, $R''(0)=0$, moreover, $R(t)=(1+ht)^\alpha$ holds for $t\ge 1$, here $\alpha\in(0,1)$ and $h>0$ are fixed constants. As in [19], we suppose that the movement of gases in $\Omega=\{(t,x):t\ge 0,\ |x|=\sqrt{x_1^2+x_2^2+x_3^2}\le R(t)\}$ is described by 3-D compressible barotropic Navier-Stokes equations:

$$\rho_t+div(\rho u)=0, \tag{1.1a}$$

$$\rho u_t+\rho u\cdot\nabla u+\nabla P(\rho)=\mu\Delta u+(\mu+\lambda)\nabla divu, \tag{1.1b}$$

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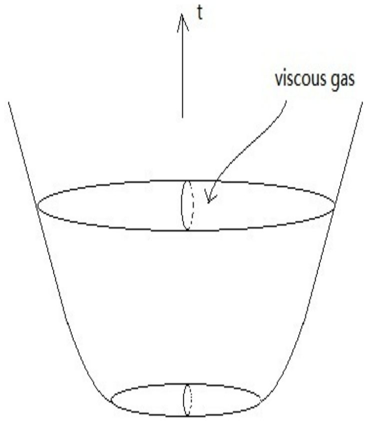

Figure 1: A viscous flow in a 3-D slowly expanded ball.

where $\rho\geq 0$ is the density, $u=(u_1,u_2,u_3)$ is the velocity, $\mu>0$ and λ are the first and second viscosity coefficient respectively, $\mu+\frac{2}{3}\lambda>0$ holds, and the state equation is $P(\rho)=\rho^\gamma$ with $\gamma>1$.

By the physical property for the viscous flow, as in [19], one can naturally pose the following initial-boundary conditions for Eqs. (1.1a)-(1.1b)

$$\begin{cases} \rho(0,x)=\rho_0(x), \quad u(0,x)=u_0(x), & \text{for } x\in S_0, \\ u(t,x)=\dfrac{R'(t)x}{R(t)}, & \text{for } (t,x)\in\partial\Omega, \end{cases} \tag{1.2}$$

where $\rho_0(x)\in H^3(S_0)$, $u_0(x)\in H_0^3(S_0)$, $\rho_0(x)>0$ for $x\in S_0$, and $\partial\Omega=\{(t,x):t\geq 0,\ |x|=R(t)\}$. For Eqs. (1.1a)-(1.1b) together with (1.2), completely similar to the proof of Theorem 2.1 in [19], we can obtain a local existence result as follows:

Theorem 1.1. *If $\rho_0(x)\in H^3(S_0)$, $\nabla\rho_0(x)\in H_0^1(S_0)$, $u_0(x)\in H_0^3(S_0)$, and $R(t)=(1+ht)^\alpha$ for $t\geq 1$, then there exist a constant $h_0>0$ and a small constant $\varepsilon_0>0$ depending only on h_0 and α such that when*

$$\sup_{0\leq t\leq 1, 1\leq k\leq 4}|R^{(k)}(t)|+\|\rho_0(x)-1\|_{H^3(S_0)}+\|u_0(x)\|_{H^3(S_0)}<\varepsilon_0 \quad \textit{and} \quad 0<h<h_0,$$

there exists some constant $T_*>1$ such that Eqs. (1.1a)-(1.1b) *with* (1.2) *have a unique local solution (ρ,u) which satisfies*

$$\begin{cases} \rho\in C([0,T_*], \quad H^3(S_t))\cap C^1([0,T_*],H^2(S_t)), \\ u\in C([0,T_*], \quad H_0^1(S_t)\cap H^3(S_t))\cap C^1([0,T_*],H_0^1(S_t))\cap L^2([0,T_*],H^4(S_t)). \end{cases}$$

Moreover, $\rho(t,x)\geq C>0$ holds for $(t,x)\in[0,T^*]\times S_t$, and

$$\|\rho-1\|_{C([0,T_*],H^3(S_t))}+\|\rho_t\|_{C([0,T_*],H^3(S_t))}+\|u\|_{C([0,T_*],H_0^1(S_t)\cap H^3(S_t))}+\|u_t\|_{C([0,T_*],H_0^1(S_t))}\leq C\varepsilon.$$