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Numerical Solution of Stochastic Ito-Volterra Integral Equations using Haar Wavelets

Fakhrodin Mohammadi*

Department of Mathematics, Hormozgan University, P. O. Box 3995, Bandarabbas, Iran.

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Abstract. This paper presents a computational method for solving stochastic Ito-Volterra integral equations. First, Haar wavelets and their properties are employed to derive a general procedure for forming the stochastic operational matrix of Haar wavelets. Then, application of this stochastic operational matrix for solving stochastic Ito-Volterra integral equations is explained. The convergence and error analysis of the proposed method are investigated. Finally, the efficiency of the presented method is confirmed by some examples.

AMS subject classifications: 65T60; 60H20; 45L05

Key words: Ito integral, Stochastic Ito-Volterra integral equations, Stochastic operational matrix, Haar wavelets, Error analysis.

1. Introduction

Stochastic functional equations (SFEs) are becoming increasingly important due to their application for modelling stochastic phenomena in different fields, e.g. biology, chemistry, epidemiology, mechanics, microelectronics, economics, and finance. The behavior of dynamical systems in these fields are often dependent on a noise source and a Gaussian white noise, governed by certain probability laws, so that modeling such phenomena naturally requires the use of various stochastic differential equations or, in more complicated cases, stochastic Volterra integral equations and stochastic integro-differential equations [1–5].

Similar to the difficulty of the deterministic functional equations, we have difficulty in finding the solution of stochastic functional equations. As analytic solutions of these equations are not available in many cases, numerical approximation becomes a practical way to face this difficulty. In previous works various numerical methods have been used for approximate the solution of SFEs. Here we only mention Kloeden and Platen [1], Oksendal [2], Maleknejad *et al.* [3,4], Cortes *et al.* [5,6], Murge *et al.* [7], Khodabin *et al.* [8,9], Zhang [10,11], Jankovic [12] and Heydari *et al.* [13].

*Corresponding author. *Email address:* `f.mohammadi62@hotmail.com` (F. Mohammadi)

Recently, different orthogonal basis functions, such as block pulse functions, Walsh functions, Fourier series, orthogonal polynomials and wavelets, were used to estimate solutions of functional equations. As a powerful tool, wavelets have been extensively used in signal processing, numerical analysis, and many other areas. Wavelets permit the accurate representation of a variety of functions and operators [14–17]. Haar wavelets have been widely applied in system analysis, system identification, optimal control and numerical solution of integral and differential equations [18, 19]. In this paper we consider the following stochastic Ito-Volterra integral equation

$$X(t) = f(t) + \int_0^t k_1(s,t)X(s)ds + \int_0^t k_2(s,t)X(s)dB(s), \quad t \in [0,T), \tag{1.1}$$

where $X(t)$, $f(t)$, $k_1(s,t)$ and $k_2(s,t)$, for $s,t \in [0,T)$, are the stochastic processes defined on the same probability space (Ω, F, P), and $X(t)$ is unknown. Also $B(t)$ is a Brownian motion process and $\int_0^t k_1(s,t)X(s)dB(s)$ is the Ito integral [2, 20]. In order to solving this stochastic Ito-Volterra integral equation we first derive the Haar wavelets stochastic integration operational matrix. Then the stochastic operational matrix for Haar wavelets along with Haar wavelets basis are used to derive a numerical solution. Convergence and error analysis of the proposed method are also investigated. Numerical results show efficiency of the proposed method.

This paper is organized as follows: In Section 2 some basic properties of the Haar wavelets are described. In Section 3 stochastic operational matrix for Haar wavelets and a general procedure for deriving this matrix are introduced. In Section 4 a computational method based on the stochastic operational matrix of Haar wavelets are proposed for solving stochastic Ito-Volterra integral equations. In Section 5 Convergence and error analysis of the proposed method are also investigated. Some numerical examples are presented in Section 6. Finally, a conclusion is given in Section 7.

2. Haar wavelets and Block pulse functions

In this section we describe some basic properties of the Haar wavelets. For this purpose we first introduce the Block pulse functions (BPFs), function approximation by BPFs and their operational matrices. Then the relations between Haar wavelets and BPFs are investigated. Finally, we derive some important formulas for Haar wavelets that are useful for the next sections.

2.1. Block pulse functions

BPFs have been studied by many authors and applied for solving different problems. In this section we recall definition and some properties of the BPFs [3, 4, 21, 22].

The m-set of BPFs are defined as

$$b_i(t) = \begin{cases} 1 & (i-1)h \le t < ih, \\ 0 & \text{otherwise}, \end{cases} \tag{2.1}$$