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A Mixed Analytical/Numerical Method for Velocity and Heat Transfer of Laminar Power-Law Fluids

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Abstract. This paper presents a relatively simple numerical method to investigate the flow and heat transfer of laminar power-law fluids over a semi-infinite plate in the presence of viscous dissipation and anisotropy radiation. On one hand, unlike most classical works, the effects of power-law viscosity on velocity and temperature fields are taken into account when both the dynamic viscosity and the thermal diffusivity vary as a power-law function. On the other hand, boundary layer equations are derived by Taylor expansion, and a mixed analytical/numerical method (a pseudo-similarity method) is proposed to effectively solve the boundary layer equations. This method has been justified by comparing its results with those of the original governing equations obtained by a finite element method. These results agree very well especially when the Reynolds number is large. We also observe that the robustness and accuracy of the algorithm are better when thermal boundary layer is thinner than velocity boundary layer.

AMS subject classifications: 74A15, 74S05, 74S30
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1. Introduction

For the last decade, the drag force behavior and energy transport behavior of non-Newtonian fluids have received considerable attention due to its wide engineering ap-

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plications, such as oil recovery, heat exchangers, and material techniques [1–3]. Non-Newtonian fluids do not exhibit the linear relationship between stress and the rate of strain. The flow and heat transfer of these fluids have been solved by a number of means with their constitutive equations varying greatly in complexity [4–6]. Among these methods, the so-called similarity transformation technique gained much attention. This method can deal with varying types of non-Newtonian fluids in different boundary conditions. For example, the laminar visco-elastic fluid flow immersed in a porous medium over a linearly stretching sheet was considered by Prasad *et al*. [7] using a similarity transformation. And, it can also be adopted in investigation in power-law fluids. Some important works included, but not limited, the followings: Y*ü*r*ü*soy [8] studied flow of a thin fluid film of a power-law caused by stretching of surface using a similarity transformation. Another similarity analysis was performed by Cheng [9] to study the steady natural convection boundary layer flow in porous media saturated with power-law fluids under mixed thermal boundary conditions. Later, Tai *et al*. [10] numerically studied the combined laminar free convection flow with thermal radiation and mass transfer of non-Newtonian power-law fluids along a vertical plate within a porous medium by applying a similarity transformation. Also, Ming *et al*. [11] dealt with the steady flow and heat transfer of a viscous incompressible power-law fluid over a rotating infinite disk by the similarity transformation assuming that the thermal conductivity follows the same power-law as the viscosity.

However, similarity transformation technique is not applicable to all problems raised from variety of non-Newtonian fluids. Note that local non-similarity method was used to obtain pseudo-similarity solutions when dealing with non-Newtonian flows. Massoudi [12] obtained local non-similarity solutions for the flow of a non-Newtonian fluid over a wedge. Wang *et al*. [13] concerned the natural convection of non-Newtonian power-law fluids with or without yield stress in a fluid-saturated porous medium using the fourth-order Runge-Kutta scheme method and shooting method to obtain the local non-similarity solutions. Meanwhile, local similarity solution was found in the case that the boundary-layer flow of a viscoelastic fluid of the second-grade type over a rigid continuous plate in [14]. Olagunju [15] studied the boundary layer flow past a semi-infinite plate of a viscoelastic fluid governed by the FENE-P model to obtain the local similarity solution. Kairi *et al*. [16] investigated the influence of viscous dissipation and Soret effect on natural convection heat and mass transfer from vertical cone in a non-Darcy porous media saturated with non-Newtonian fluid by local non-similarity method. These researches working on local non-similarity method of predecessors inspire our work. We also proposed a modified similarity transformation technique to cope with non-Newtonian problems which cannot be solved by traditional methods.

The present work focuses on the significance of the rheology of power-law fluids over a semi-infinite plate in the presence of viscous dissipation and y-direction radiation. The laminar forced convection problem is considered when both the kinematic viscosity and the thermal diffusivity vary as a power-law function. The governing equations near the boundary plate can be approximated by boundary layer equations. In the region away from boundary layer, the governing equations reduce to the incom-