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The Immersed Interface Method for Simulating Two-Fluid Flows

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Abstract. We develop the immersed interface method (IIM) to simulate a two-fluid flow of two immiscible fluids with different density and viscosity. Due to the surface tension and the discontinuous fluid properties, the two-fluid flow has nonsmooth velocity and discontinuous pressure across the moving sharp interface separating the two fluids. The IIM computes the flow on a fixed Cartesian grid by incorporating into numerical schemes the necessary jump conditions induced by the interface. We present how to compute these necessary jump conditions from the analytical principal jump conditions derived in [Xu, *DCDS*, Supplement 2009, pp. 838-845]. We test our method on some canonical two-fluid flows. The results demonstrate that the method can handle large density and viscosity ratios, is second-order accurate in the infinity norm, and conserves mass inside a closed interface.

AMS subject classifications: 76M20, 65M06, 35Q35

Key words: Immersed interface method, two-fluid flows, jump conditions, augmented variable approach, singular force, Cartesian grid methods.

1. Introduction

Many natural and industrial processes involve the flow of two immiscible fluids. Examples include rise of steam in boiler tubes, bubbles in oil wells, ocean waves, geysers and sprays. Direct numerical simulations can potentially increase the understanding of such flows. There are several difficulties in the direct numerical simulation of a two-fluid flow. The interface separating the two fluids is extremely thin, leading to the discontinuities of fluid density and viscosity in the flow field. The existence of surface tension would induce a pressure jump across the interface as well. Other factors such as high density and viscosity ratios, phase transition, topological changes, and a vast

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range of time and length scales make the development of a robust numerical method even more challenging [8].

In recent years, different numerical methods have been proposed to solve the governing Navier-Stokes equations for two-fluid flows, and each of them has its own strengths and weaknesses. These methods can be classified into two groups: Lagrangian methods that modify the grid to match the interface location, and Eulerian methods that extract the interface location from a fixed grid.

In a Lagrangian method, the computational mesh moves and distorts with an interface. A Lagrangian method permits an interface to be specifically delineated and precisely followed, and it allows interfacial conditions to be easily applied [6]. Lagrangian methods are successful for small interface deformations [30]. However, they have difficulties when interface undergoes large deformations to require re-meshing [27]. Some examples of Lagrangian methods can be found in [7, 21, 27].

In an Eulerian method, an interface moves through a fixed grid and its position is computed at each time step. The two main approaches to follow the interface motion are interface capturing and interface tracking. With interface capturing, the interface is implicitly captured by a contour of a scalar function. Some popular examples of this kind are the volume of fluid (VOF) method [9] and the level set method [23, 28]. In the VOF method, the location of an interface is determined by the volume fraction occupied by each fluid in each computational cell. In the level set method, an interface is represented as a zero set of an auxiliary scalar function (level set function). The signed distance function is commonly used as the scalar function. An interface tracking method uses a set of Lagrangian points to mark and track an interface. The interface is treated with either finite thickness or zero thickness. Examples of interface tracking methods include the front-tracking method [33, 34] and the ghost fluid method (GFM) [5, 15]. In the front-tracking method [33, 34], a two-fluid flow is treated and solved as one system with the delta function formulation, and the interface is smoothed by the discrete approximation of the delta function. The GFM [15] eliminates the numerical smearing prevalent in the delta function formulation and treats the interface in a sharp fashion. Its basic idea is to extrapolate the solution in each fluid onto fictitious ghost nodes located in the other fluid, and then solve the governing equations in both fluids separately [40].

In an attempt to overcome some of the limitations of the above methods, there has been some hybrid methods which exploit the best features of different methods. Some examples of hybrid methods include the level-set/volume-of-fluid methods [31], the particle level set method [4], the marker/volume-of-fluid methods [1], the level-contour front tracking methods [29], and the level-set/immersed boundary method [39].

The immersed interface method (IIM) [20] was initially proposed by LeVeque and Li [17] to improve the accuracy of Peskin's immersed boundary (IB) method [24, 25]. The IIM differs from the IB method in the treatment of the singular force appearing in the delta function formulation of an interface problem. The IIM can capture the jumps of a solution and its derivative by incorporating them directly into numerical schemes.