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Numerical Method for Singularly Perturbed Third Order Ordinary Differential Equations of Convection-Diffusion Type

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Abstract. In this paper, we have proposed a numerical method for Singularly Perturbed Boundary Value Problems (SPBVPs) of convection-diffusion type of third order Ordinary Differential Equations (ODEs) in which the SPBVP is reduced into a weakly coupled system of two ODEs subject to suitable initial and boundary conditions. The numerical method combines boundary value technique, asymptotic expansion approximation, shooting method and finite difference scheme. In order to get a numerical solution for the derivative of the solution, the domain is divided into two regions namely inner region and outer region. The shooting method is applied to the inner region while standard finite difference scheme (FD) is applied for the outer region. Necessary error estimates are derived for the method. Computational efficiency and accuracy are verified through numerical examples. The method is easy to implement and suitable for parallel computing.

AMS subject classifications: 65L10

Key words: Singularly perturbed problems, third order ordinary differential equations, boundary value technique, asymptotic expansion approximation, shooting method, finite difference scheme, parallel computation.

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1. Introduction

Singular Perturbation Problems (SPPs) arise frequently in many fields like geophysical dynamics, oceanic and atmospheric circulation, chemical reactions, etc. The presence of small parameter(s) in these problems prevent(s) us from obtaining satisfactory numerical solutions using classical numerical methods. It is a well known fact that the solutions of the SPPs have multi-scale character. That is there are thin transition layers where the solution can jump abruptly, while away from the layers the solution

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behave regularly and varies slowly. Such problems have been investigated by many researchers. The existence and uniqueness of such problems are discussed in [4, 20]. In recent years a variety of numerical methods are available in the literature to solve SPBVPs for second order ODEs. For details,one may refer the survey article [8], but for higher order equations only few results are reported in the literature.

Analytical treatment of SPBVPs for the higher order non-linear ODEs which have important applications in fluid dynamics is discussed in [3, 10, 16, 20, 29]. Niederdrenk and Yserentant [16] have considered convection-diffusion type problems and derived conditions for the uniform stability of the discrete and continuous problems. Gartland [3] has shown that the uniform stability of the discrete BVP follows from the uniform stability of the discrete IVP and uniform consistency of the scheme. In [20], an iterative method is described.

Feckan [10] has considered higher order problems and his works are based on the non-linear analysis involving fixed point theory, Leray-Schauder theory, etc. In fact, Howes [6] has considered the higher order problems and discussed the existence, uniqueness and asymptotic estimates of the solution. In [20,29], a FEM for convection-reaction type problems is described.

As far as author's knowledge goes only few results are reported in the case of third order differential equations. Zhao [27] has considered a more general class of third order non-linear SPBVPs and discussed the existence, uniqueness of the solution and obtained asymptotic estimates using the theory of differential inequalities. In fact Zhao [28] has derived results on third order non-linear SPPs using differential inequality theorems. Howes [5] has considered class of third order SPBVP and discussed the existence, uniqueness and asymptotic behaviour of the solution. Roberts [18] has suggested a method of finding approximate solutions for third order SPODEs. Valarmathi [21–24] have suggested methods of finding approximate solutions for third order SPBVPs.

Following the Boundary Value Technique (BVT) of Roberts [18], Vigo-Aguiar [26], Valarmathi [21] and using the basic idea underlying the method suggested in Jayakumar [7] and Natesan [12] we in the present paper, suggest a new computational method which makes use of the zero order asymptotic expansion approximation, BVT and shooting method to obtain a numerical solution for the derivative of SPBVPs for third order ODEs of convection-diffusion type of the form:

$$\varepsilon y'''(x) + a(x)y''(x) - b(x)y'(x) - c(x)y(x) = f(x), \quad x \in \Omega, \tag{1.1}$$

$$y(0) = p, \quad -y''(0) = q, \quad y'(1) - y''(1) = r, \tag{1.2}$$

where $0 < \varepsilon \ll 1$, $a(x)$, $b(x)$, $c(x)$ are sufficiently smooth functions satisfying the following conditions:

$$a(x) \geq \alpha, \quad \alpha > 0, \tag{1.3}$$

$$b(x) > 0, \tag{1.4}$$

$$0 \geq c(x) \geq -\gamma, \quad \gamma > 0, \tag{1.5}$$

$$\alpha > \gamma, \tag{1.6}$$