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A Method for Solving the Inverse Scattering Problem for Shape and Impedance of Crack

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Abstract. The inverse problem considered in this paper is to determine the shape and the impedance of crack from a knowledge of the time-harmonic incident field and the corresponding far field pattern of the scattered waves in two-dimension. The combined single- and double-layer potential is used to approach the scattered waves. As an important feature, this method does not require the solution of u and $\partial u/\partial \nu$ at each iteration. An approximate method is presented and the convergence of this method is proven. Numerical examples are given to show that this method is both accurate and simple to use.

AMS subject classifications: 34L25, 81U40

Key words: Impedance boundary condition, Helmholtz equation, inverse scattering problem.

1. Introduction

The inverse scattering problem for electromagnetic time-harmonic plane wave by very thin obstacles has been considered in a series of papers [1–3]. Among these papers, the Dirichlet and Neumann crack problem has been solved. In the paper [4], the inverse problem considered is to determine the shape and the impedance of an obstacle from a knowledge of the time-harmonic incident field and the phase and amplitude of the far field pattern of the scattered wave in two-dimension. In this paper we are interested in numerical methods for determining the shape and impedance for crack from the knowledge of the incident field and the scattered field of the far field pattern. The difference is that the closed boundary curve is considered in paper [4] but a non-intersecting arc is considered in this paper. And the combined potential is put forward.

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In comparison with [1], our method considers the impedance problem. In paper [5], the same problem is considered. But in this paper, using the combined single and double layer potentials to approach the scattered field u^s, the problem is changed to a minimization problem. Furthermore, our reconstructions do not require the solution of the function u and its normal derivative $\partial u/\partial \nu$ at each iteration step and only require the nonzero initials of φ, Γ, λ. We approximate the functions ψ, z and λ by finite trigonometric series.

Let $\Gamma \subset \mathbb{R}^2$ be a non-intersecting C^3-smooth arc, i.e.,

$$\Gamma = \{z(t) : t \in [-1,1],\ z \in C^3[-1,1] \ \text{ and } \ |z'(t)| \neq 0,\ \ \forall t \in [-1,1]\}.$$

By z_1, z_{-1} we denote the two end points $z_1 := z(1)$ and $z_{-1} := z(-1)$ of Γ and set $\Gamma_0 := \Gamma \setminus \{z_{-1}, z_1\}$. Assuming an orientation for Γ from z_{-1} to z_1, by Γ_+ and Γ_- we denote the left- and right-hand sides of Γ, respectively, and by ν the unit normal vector to Γ directed towards Γ_+. Let the incident field u^i be given by $u^i(x) = \exp[ikx \cdot d]$, where $k > 0$ is the wave number and d is a fixed unit vector. The direct scattering problem consists of finding the total field $u = u^i + u^s$ such that both the Helmholtz equation

$$\Delta u + k^2 u = 0 \quad \text{in } \ \mathbb{R}^2 \backslash \Gamma \tag{1.1}$$

and the impedance boundary condition

$$\frac{\partial u_\pm}{\partial \nu} \pm \mathrm{i}k\lambda u_\pm = 0 \quad \text{on } \ \Gamma_0 \tag{1.2}$$

are satisfied. To ensure uniqueness, the Sommerfeld radiation condition

$$\lim_{r\to\infty} \sqrt{r} \left\{ \frac{\partial u^s}{\partial r} - \mathrm{i}ku^s \right\} = 0, \quad r = |x| \tag{1.3}$$

is imposed uniformly for all directions.

The radiation condition (1.3) ensures an behavior of the form

$$u^s(x) = \frac{e^{\mathrm{i}k|x|}}{\sqrt{|x|}} \left(u_\infty(\hat{x}) + \mathcal{O}\left(\frac{1}{|x|}\right) \right), \quad |x| \to \infty \tag{1.4}$$

uniformly for all directions $\hat{x} = x/|x|$ (see [6]). The amplitude factor u_∞ is known as the far field pattern of the scattered wave u^s and defined in the unit circle $\Omega \subset \mathbb{R}^2$. The inverse problem we are concerned with is to determine the impedance and the shape of crack Γ from a knowledge of the far field pattern u_∞ for the incident wave u^i.

For the problem (1.1)-(1.3), there exists the following theorem.

Theorem 1.1 (see [7]). *The impedance crack problem has at most one solution.*