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An Adaptive Strategy for the Restoration of Textured Images using Fractional Order Regularization

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Abstract. Total variation regularization has good performance in noise removal and edge preservation but lacks in texture restoration. Here we present a texture-preserving strategy to restore images contaminated by blur and noise. According to a texture detection strategy, we apply spatially adaptive fractional order diffusion. A fast algorithm based on the half-quadratic technique is used to minimize the resulting objective function. Numerical results show the effectiveness of our strategy.

AMS subject classifications: 65F10, 65F22, 65K10
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1. Introduction

Noise reduction and deblurring are usually used in a pre-processing stage in image restoration to improve image quality. In this paper, we focus on texture preserving restoration of images corrupted by additive noise and spatially-invariant Gaussian blur. The most common image degradation model, where the observed data $f \in \mathbb{R}^{n^2}$ is related to the underlying $n \times n$ image rearranged into a vector $u \in \mathbb{R}^{n^2}$, is

$$f = Bu + e, \tag{1.1}$$

where $e \in \mathbb{R}^{n^2}$ accounts for the random perturbations due to noise and B is a $n^2 \times n^2$ matrix representing the linear blur operator.

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It is well known that restoring the image u is a very ill-conditioned problem and a regularization method should be used. A popular approach determines an approximation of u as the solution of the minimization problem

$$\min_u \left\{ \frac{1}{p} \|Bu - f\|_p^p + \frac{\lambda}{q} \|A(u)\|_q^q \right\}, \tag{1.2}$$

where A is a regularization operator and λ is a positive regularization parameter that controls the trade-off between the data fitting term and the regularization term [13,25,27]. For $p = 2$ and $q = 2$, we get the classical Tikhonov regularization [11,13]. This approach enforces smoothness of the solution and suppresses noise by penalizing high-frequency components, thus also image edges can be smoothed out in the process.

Numerous regularization approaches and advances numerical methods have been proposed in the literature to better preserve edges, including alternating minimization algorithms [1], multilevel approaches [16], non-local means filters [4].

A very popular choice in the literature for regularization is based on the total variation (TV) norm. Total variation minimization was originally introduced for noise reduction [7,25] and has also been used for image deblurring [14] and super-resolution image reconstruction [17]. The TV regularization (ℓ_2-TV) is obtained from (1.2) by setting $p = 2$, $q = 1$ and $A(u)$ the gradient magnitude of u. If we let $\nabla u_i := (G_{x,i}u, G_{y,i}u)^T$, with $G_{x,i}$, $G_{y,i}$ representing the ith rows of the x and y-directional finite difference operators G_x, G_y, respectively, then the regularization term is defined by the TV-norm

$$\|u\|_{TV} = \|A(u)\|_1 := \sum_{i=1}^{n^2} \sqrt{(G_{x,i}u)^2 + (G_{y,i}u)^2}.$$

The distinctive feature of TV regularization is that image edges can be preserved, but the restoration can present staircase effects.

A variant of the ℓ_2-TV regularization is the ℓ_1-TV regularization which is obtained from (1.2) by replacing the ℓ_2 norm in the data-fitting term by ℓ_1 norm:

$$\min_u \{ \|Bu - f\|_1 + \lambda \|u\|_{TV} \}, \tag{1.3}$$

see, e.g., [5,26–29] for discussions on this model. This model has a number of advantages, including superior performance with non-Gaussian noise such as impulse noise, see [20]. However, it is well known that the ℓ_1-TV regularization has problems in preserving textures, see [6,28].

In image denoising, recent works have dealt with this drawback mainly by two different strategies. In [10] the ℓ_2-TV model is adopted using a spatially variant regularizing parameter λ, selected according to local variance measures. In [2], a fractional order anisotropic diffusion model is introduced, which leads to a "natural interpolation" between the Perona-Malik equations [22] and fourth-order anisotropic diffusion equations [15]. An adaptive fractional-order multi-scale model is proposed in [31,32], where the model is applied to noise removal and the texture is detected by a variant of the strategy in [10].