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An Asymptotic/Numerical Study for the Micropolar Fluid Asymmetric Flow in a Porous Channel with Orthogonally Moving Walls

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Dedicated to Professor Xiaoqing Jin on the occasion of his 60th birthday

Abstract. The two-dimensional, unsteady, viscous, incompressible and asymmetric flow of a micropolar fluid in a uniformly porous channel with expanding or contracting walls is investigated. We assume that the channel walls have different permeability and that the flow is driven by uniformly suction at the lower and upper walls. The corresponding governing equations of motion are reduced to nonlinear coupled ordinary difference equations by a similarity transformation. For the most difficult high Reynolds number case, we construct an asymptotic solution by the method of boundary layer correction. There exist boundary layers near the both walls. Furthermore, this asymptotic results can also be used to validate the numerical method over a range of Reynolds numbers. Finally, the influence of Reynolds number R , asymmetric parameter a, expansion ratio α and micropolar parameter n on the flow is discussed numerically. The results show that the streamwise and microrotation velocity are sensitive to the parameters.

AMS subject classifications: 76M45, 76D10

Key words: Micropolar fluid, asymptotic solution, asymmetric flow, porous channel flow.

1. Introduction

Laminar flows in permeable walls have been extensively studied by researchers during the past several decades due to a variety of applications in the fields of filtration, contaminant transports in aquifers and fractures, fluid flow in the biological organisms and air circulation in the respiratory system. The blood flow especially in the vessels has significant effect on the human body and its dynamics is closely related to human health.

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Take atherosclerosis as an example in [1], the intimal thickening of stenos artery influence the flow behavior of blood. Blood supply and oxygen to the brain will reduced once serve stenosis suppresses the blood flow. Hence, investigations of fluid transport and flow through contracting and expanding vessels will provide a full understanding of the development of these diseases, assist bioengineers to the design and construct improved artificial organs and also help the treatment of vascular diseases [1].

Uchida and Aoki [2] established a mathematical model about the Newtonian fluid flow in a porous tube with expansion or contraction cross section. It is the first to introduce the expansion ratio to investigate the influence of a moving channel to the flow. After taking into account of the wall motion, Majdalani and Zhou [3] studied the porous channel flow (with expanding or contracting walls) and presented a similarity asymptotic solution by assuming that the expansion ratio is a constant for slowly expanding and small injection (or suction) case. Along this line, Majdalani and Zhou [4] investigated the large injection and suction cases and obtained the asymptotic solutions which were verified by numerical results. Dauenhauer and Majdalani [5] employed a Runge-Kutta method coupled with a rapidly converging shooting technique to study a modest range of R and wall expansion ratios numerically and their influence on the flow. Xu et al. [6] obtained accurate series approximations using the homotopy analysis method (HAM). The above studies were carried out for symmetric flows. Asymmetric flows were of an interesting topic as well. The asymmetric laminar steady flow caused by different wall permeability may be traced back to Proudman [7] who only presented the asymmetric problem. Then, Terrill et al. [8–10] extended Proudman's work and obtained asymptotic solutions using the matched expansion method for large injection, large suction and mixed cases. Cox [11, 12] considered the both problem of steady and unsteady asymmetric flows in a channel with only porous channel. But, the studies on asymmetric problem did not consider any wall motion.

All works mentioned above consider only Newtonian fluid, however, blood and many other flows exhibit non-Newtonian characteristics. Hence, the theory of microfluids was established by Eringen [13] initially in 1964. The micropolar fluids exhibit microscopic effects and are inflenced by the spin inertia. Eringen [14] again introduced the concept of micorofluids to describe suspensions of neutrally buoyant deformable particles in viscous fluid. Such fluid can be applied to describe normal human blood and polymeric suspensions and have applications in physiological and engineering problems [15, 21]. Afterwards, Mekheimer and Kot [22] investigated the micropolar fluid for blood flow through a tapered artery with a stenosis. Agarwal and Dhanapal [15] discussed steady laminar flow of a micropolar fluid through porous walls of different permeability numerically. Ashraf et al. [16] considered the asymmetric flow of a micropolar fluid in a porous channel. They transformed the governing equations of motion into ordinary difference equations by a similarity transformation, investigated the injection case numerically and discussed the influence of asymmetric parameter on the flow velocities and microrotation. The numerical solutions of a micropolar fluid flow between two porous coaxial disks were given by Kamal et al. [17]. For the porous channel problem, Si et al. [23–25] began to take into account of the wall motion and HAM was used to obtain an appropriate solution for the flow velocity and microrotation. But, they only considered the case that the flow is symmetric about the