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Superlinear Convergence of a Smooth Approximation Method for Mathematical Programs with Nonlinear Complementarity Constraints

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Abstract. Mathematical programs with complementarity constraints (MPCC) is an important subclass of MPEC. It is a natural way to solve MPCC by constructing a suitable approximation of the primal problem. In this paper, we propose a new smoothing method for MPCC by using the aggregation technique. A new SQP algorithm for solving the MPCC problem is presented. At each iteration, the master direction is computed by solving a quadratic program, and the revised direction for avoiding the Maratos effect is generated by an explicit formula. As the non-degeneracy condition holds and the smoothing parameter tends to zero, the proposed SQP algorithm converges globally to an S-stationary point of the MPEC problem, its convergence rate is superlinear. Some preliminary numerical results are reported.

AMS subject classifications: 90C30, 65K05

Key words: Mathematical programs with complementarity constraints, nonlinear complementarity constraints, aggregation technique, S-stationary point, global convergence, super-linear convergence.

1. Introduction

Mathematical programs with equilibrium constraints (MPEC) is an optimization problem whose constraints include variational inequalities or complementarity system. It forms a relatively new and interesting class of optimization problems, which have found many applications in engineering and economics, we refer to [10] for an extensive bibliography on this topic and its applications. In this paper, we consider an important subclass of the MPEC problem, which is called mathematical programs with complementarity constraints (MPCC):

$$\begin{array}{ll} \min & f(x,y) \\ \text{s.t.} & 0 \le F(x,y) \perp y \ge 0, \end{array} \tag{1.1}$$

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where $f : R^{n+m} \to R, F : R^{n+m} \to R^m$ are continuously differentiable functions, and $w \perp y$ indicates orthogonality of any vectors $w, y \in R^m$. The system $0 \leq F(x,y) \perp y \geq 0$ is said to be the lower-level or equilibrium constraints.

The major difficulty in solving problem (1.1) is that its constraint fails to satisfy the standard Mangasarian-Fromovitz constraints qualification at any feasible point because of the existence of the complementarity constraint, see [1] and [13]. So the theory for nonlinear programming can not be directly applied to problem (1.1), hence the standard methods are not guaranteed to solve such problem. In recent years, optimal conditions and various stationarity concepts were deeply studied by some authors, see [10, 13, 15]. For example, Luo et al. [10] provided a comprehensive study on the MPEC, such as the exact penalization theory, stationarity conditions. Scheel and Scholtes [13] made an excellent clarification on these concepts and elucidated their connections. More recently, Qi et al. [12] investigated the differentiable properties of the aggregation function, and used it to propose a smoothing method for nonlinear complementarity problems. Jiang and Ralph [8] proposed two smooth SQP methods for MPEC. Global convergence of the methods depend on the lower level strict complementarity condition amongst some conditions, such as the LICQ or MFCQ. Fukushima and Tseng [5] proposed an ε-active set algorithm, they used a sequence of SSNPs based on an ε-active set to approach the discussed MPCC. Under a uniform LICQ on the ε-feasible set, this algorithm generates iterates whose cluster points are B-stationary points of the problem. However, the proof has a gap and shows only that each cluster point is an M-stationary point. Subsequently, Fukushima and Tseng [6] discussed this gap and a modified algorithm that achieves B-stationarity under an additional error bound condition. Tao [14] proposed a class of smoothing methods for MPCC, they used an available probability density function to obtain a corresponding approximation of the original problem. Under some conditions, the MPCC-LICQ holds for the class of smooth methods. However, the methods of [5, 6, 8, 12, 14] do not adopt any technique to avoid the Maratos effect, they only possess global convergence.

Motivated by the ideas of [8, 12, 14], we present a new smoothing SQP algorithm for the problem (1.1). Some notable features of the new algorithm are as follows:at each iteration, the master direction is computed by solving a quadratic program (QP) which only includes equality constraints, the form of QP is different from in [8]. The revised direction for avoiding the Maratos effect is obtained by an explicit formula. The proposed algorithm possesses not only global convergence, but also super-linear convergence.

The structure of this paper is organized as follows: In Section 2, some known results are restated. In Section 3, the algorithm is proposed. In Section 4, we show that the algorithm is globally convergent. Super-linear convergence rate is proved in Section 5, and finally some preliminary numerical results are reported in Section 6.

Throughout this paper, we use the following notations:

$$
\begin{aligned}
&z = (x, y, w), \quad p = (x, y), \quad q = (y, w), \\
&dp = (dx, dy), \quad dq = (dy, dw), \quad dz = (dx, dy, dw), \\
&p^k = (x^k, y^k), \quad q^k = (y^k, w^k), \quad dz^k = (dx^k, dy^k, dw^k).
\end{aligned}
$$