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Cooperative Sensing and Distributed Control of a Diffusion Process Using Centroidal Voronoi Tessellations

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Abstract. This paper considers how to use a group of robots to sense and control a diffusion process. The diffusion process is modeled by a partial differential equation (PDE), which is a both spatially and temporally variant system. The robots can serve as mobile sensors, actuators, or both. Centroidal Voronoi Tessellations based coverage control algorithm is proposed for the cooperative sensing task. For the diffusion control problem, this paper considers spraying control via a group of networked mobile robots equipped with chemical neutralizers, known as smart mobile sprayers or actuators, in a domain of interest having static mesh sensor network for concentration sensing. This paper also introduces the information sharing and consensus strategy when using centroidal Voronoi tessellations algorithm to control a diffusion process. The information is shared not only on where to spray but also on how much to spray among the mobile actuators. Benefits from using CVT and information consensus seeking for sensing and control of a diffusion process are demonstrated in simulation results.

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Key words: Consensus, centroidal Voronoi tessellations, diffusion process, distributed control, mobile actuator and sensor networks.

1. Introduction

Diffusion processes like chemical/radiation leaks, oil spills etc. can have a large impact on human health and natural environment. Nowadays, technological advances in networking and MEMS (Micro-Eletro-Mechanical Systems) make it possible to employ a large number of mobile/static sensors/actuators to observe the diffusion, locate the source, and even counter-react with the harmful pollutants when the mobile spray network is used. In the past decade, many researchers looked into this topic. A swarm of mobile robots

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are used to detect chemical plume source with gradient climbing [1]; a moving diffusion source can be identified based on the parameter estimation algorithm [2]; boundary estimation and following problem is considered [3]. However, only the source information is not enough for controlling a diffusion process. Centroidal Voronoi tessellations are introduced in coverage control of a static gradient field with mobile sensor networks [4–6] and extended to a diffusing and spaying scenario [7].

Actually, the monitoring and control of a diffusion process can be viewed as an optimal sensor/actuator placement problem in a distributed system [8]. Basically, a series of desired actuator positions are generated based on centroidal Voronoi tessellations and later integrated with PID controllers for neutralizing control based on Voronoi partitions. CVT algorithm provides a non-model-based method for coverage control and diffusion control using groups of vehicles. The CVT algorithm is robust and scalable [9, 10] and it can guarantee the groups asymptotically converging to the affected area even in multiple/mobile sources application [4].

Consensus is a common agreement reached by a group as a whole. The consensus can be made on robot formation, source location tracking, task assignment, and traffic control [11, 12, 14]. Although a group of mobile actuators are used for the diffusion control [7], the communication and information aspects are not taken care of. The mobile actuator only negotiates with its neighboring sensors, not neighboring actuators/sprayers, on how much to spray and where to go. As will be known in this paper, the information sharing and interaction among neighboring actuators/sprayers in a group can have a large impact on the coordinated movements of these actuators and the resulted control performance consequently. Since the actuators are sent out for the same task, consensus is needed on both where to spray and how much to spray. The mobile actuators need to get close to the polluted area but it is not efficient to cluster, or running together densely. On the other hand, the neutralizer spraying should also be balanced since the best energy saving way is to maximize the neutralizing ability of every actuator. A new consensus algorithm is introduced and integrated into the CVT algorithm to guarantee the actuator group to converge faster towards the affected area with an improved control performance.

The remaining part of this paper is organized as follows. In Section 2, the diffusion process is modeled by a PDE equation and the diffusion control problem is formulated. In Section 3, centroidal Voronoi tessellations based optimal actuator location algorithm is briefly introduced. Section 4 is devoted to introducing the information consensus into the CVT based optimal actuator location algorithm. Simulation results and comparisons with our previous CVT algorithm are presented Section 5. Finally, conclusions and future research directions are given in Section 6.

2. Mathematical modeling and problem formulation

In this section, the PDE mathematical model of a diffusion process is introduced and the neutralizing control problem is then formulated.

Suppose a diffusion process evolves in a convex polytope Ω: $\Omega \in \mathscr{R}^2$. $\rho(x, y) : \Omega \to \mathscr{R}_+$ is used to represent the pollutant concentration over Ω. The dynamic process can be