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An hp-Version Chebyshev Spectral Collocation Method for Nonlinear Volterra Integro-Differential Equations with Weakly Singular Kernels

Hongli Jia[1], Yang Yang[2] and Zhongqing Wang[3,*]

[1] *College of Science, Donghua University, Shanghai, 200063, China*
[2] *School of Mathematical Sciences, Tongji University, Shanghai, 200092, China*
[3] *College of Science, University of Shanghai for Science and Technology, Shanghai, 200093, China*

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Abstract. This paper presents an hp-version Chebyshev spectral collocation method for nonlinear Volterra integro-differential equations with weakly singular kernels. The hp-version error bound of the collocation method under the H^1-norm is established on an arbitrary mesh. Numerical experiments demonstrate the effectiveness of the proposed method.

AMS subject classifications: 65L60, 45D05, 41A10

Key words: Chebyshev spectral collocation method, nonlinear Volterra integro-differential equations, weakly singular kernels, error analysis.

1. Introduction

Volterra integro-differential equations (VIDEs) have important applications in many branches of science, including mechanics, engineering, chemistry and so on (cf. [17, 28]). Various effective numerical methods have been proposed and analyzed for solving linear and nonlinear VIDEs, which mainly include:

(i). The h-version method, e.g., the h-version collocation [2, 11, 20], Runge-Kutta [1, 31], finite difference [26], and continuous and discontinuous Galerkin [14, 15] methods. Furthermore, we refer to the works [3, 5, 6] and the literature provided therein.

(ii). The p-version method, e.g., the p-version spectral Galerkin and collocation methods [7, 8, 12, 13, 18, 21].

(iii). The hp-version methods, e.g., the hp-version continuous Galerkin, discontinuous Galerkin, and spectral collocation methods [4, 16, 19, 25, 29, 30].

*Corresponding author. *Email address:* `zqwang@usst.edu.cn` (Z. Wang)

The hp-version method is attractive and popular, since it allows for locally varying time steps and approximation orders, and can approximate smooth solutions with possible local singularities at high algebraic or even exponential convergence rates.

In this paper, we focus on constructing the hp-version Chebyshev spectral collocation method for the following nonlinear VIDEs with weakly singular kernels:

$$\begin{cases} \dfrac{d}{dt}U(t) = f(t,U(t)) + \displaystyle\int_0^t (t-\tau)^{-\nu} G(t,\tau,U(\tau))d\tau, \qquad t \in \big(0,T\big], \\ U(0) = U_0, \end{cases} \tag{1.1}$$

where $0 < \nu < 1$, $f(t,U)$ and $G(t,\tau,U)$ are given functions, and U_0 represents the initial data.

Relatively few theoretical results are available for weakly singular VIDEs by means of hp-version collocation methods. Recently, Wang et al. [22,23] proposed a Legendre spectral collocation method for linear and nonlinear VIDEs with weakly singular kernels, and derived the hp-version error bounds of the collocation method, which motivates us to construct the Chebyshev spectral collocation method for VIDEs (1.1).

As is well known that, the Chebyshev spectral method is a very important numerical approach (cf. [27]), which permits the use of fast Chebyshev transformation (FCT) and has been applied for numerically simulating various partial differential equations. In this paper, we introduce and analyze the hp-version Chebyshev spectral collocation method for nonlinear VIDEs (1.1). The main difficulties encountered in the convergence analysis are as follows:

(a). The problem considered is nonlinear.

(b). The integral kernels are weakly singular and the solution to Eq. (1.1) is usual singular.

(c). The Chebyshev weight function is weakly singular and will destroy the symmetry of the numerical scheme.

To this end, we employ two types of polynomial interpolations, namely, the shifted Chebyshev-Gauss-Lobatto and shifted Jacobi-Gauss interpolations, and introduce their relevant properties. Accordingly, we construct the hp-version Chebyshev spectral collocation scheme and design the algorithm. We carry out a rigorous hp-version error analysis of the proposed method on an arbitrary mesh, and present some numerical experiments to verify the theoretical results.

The remainder of this paper is arranged as follows. In Section 2, we introduce certain basic properties of the shifted Chebyshev/Jacobi polynomial interpolations and propose the hp-version Chebyshev spectral collocation method for nonlinear VIDEs (1.1). Section 3 is devoted to deriving the hp-version error bound of the Chebyshev collocation method on an arbitrary mesh. In Section 4, we present a series of numerical examples to validate our theoretical results.