Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.OA-2017-0004

Vol. **11**, No. 1, pp. 128-139
February 2018

On the Convergence of Two-Step Modulus-Based Matrix Splitting Iteration Methods for a Restricted Class of Nonlinear Complementarity Problems with H_+-Matrices

Rui Li[1,2], Yan Wang[1] and Junfeng Yin[1,*]

[1] *School of Mathematical Sciences, Tongji University, Shanghai 200092, China*
[2] *College of Mathematics Physics and Information Engineering, Jiaxing University, Zhejiang 314001, China*

Received 13 September 2016; Accepted (in revised version) 24 March 2017

Abstract. We propose the two-step modulus-based matrix splitting iteration methods for a class of nonlinear complementarity problems. The corresponding convergence theory is established when the system matrix is an H_+-matrix. Theoretical analysis gives the choice of parameter matrix involved based on the H-compatible splitting of the system matrix. Moreover, in actual implementation, the choices of iterative parameters for two-step modulus-based accelerated overrelaxation methods are studied. Numerical experiments show that the method is efficient and further verify the convergence theorems.

AMS subject classifications: 90C33, 65F10, 65F50, 65G40

Key words: Nonlinear complementarity problems, two-step modulus-based matrix splitting methods, H_+-matrix, H-compatible splitting.

1. Introduction

Given a matrix $A \in \mathbb{R}^{n\times n}$, a vector $q \in \mathbb{R}^n$ and a nonlinear mapping $f : \mathbb{R}^n \to \mathbb{R}^n$, the following nonlinear complementarity problem which aims to find n-dimensional real vectors z and w such that

$$z \geq 0, \quad w := Az + q + f(z) \geq 0, \quad z^T w = 0, \tag{1.1}$$

arises widely from many scientific computing and engineering applications, such as the network equilibrium problem, the contact problem and the free boundary problem with nonlinear source terms [1, 2]. In the free boundary problem, the function f is usually

*Corresponding author. *Email addresses:* lirui@tongji.edu.cn (R. Li), 7ywang@tongji.edu.cn (Y. Wang), yinjf@tongji.edu.cn (J. F. Yin)

referred to the nonlinear source term. Moreover, f in problem (1.1) is assumed to be a diagonal differentiable mapping [3,4], which means the ith component of f is differentiable and only in terms of z_i, i.e.,

$$f_i = f_i(z_i), \quad i = 1, 2, \cdots, n.$$

In problem (1.1), '≥ 0' is componentwise and the superscript 'T' means the transpose of a vector.

If f is a linear function, the nonlinear complementarity problem (1.1) reduces to the linear complementarity problem. In the past decades, a number of efficient iteration methods have been proposed for solving linear complementarity problems, especially when the coefficient matrix is a real positive definite matrix or an H_+-matrix. An excellent survey of the existing methods and the classification of matrices for linear complementarity problems can be found in [2]. By combining the modulus method [5–7] and the matrix splitting technique, Bai [8] presented the modulus-based matrix splitting iteration method for linear complementarity problems and obtained the convergence theorems when A is positive definite or an H_+-matrix. Further, Zhang and Ren improved the convergence condition by weakening the H-compatible splitting of an H_+-matrix to the H-splitting of it in [9]. Inspired by the work in [8], a series of modulus-based matrix splitting iteration methods were developed. For instance, the two-step modulus-based matrix splitting iteration method was proposed by Zhang and its convergence theory was proved when the system matrix is an H_+-matrix [10, 11]. By putting another parameter diagonal matrix to the fixed-point formula, Li constructed a general modulus-based matrix splitting iteration scheme in [12]. Further, Xu and Liu [13] gave a modified general modulus-based matrix splitting method by replacing a positive diagonal matrix with a nonnegative one. See [14–21] for more variants of modulus-based matrix splitting iteration methods.

For the solution of large and sparse nonlinear complementarity problem, iteration methods also attract much attention of researchers. The parallel nonlinear multisplitting relaxation method and the Newton-type method were studied in [22, 23], respectively, and only local convergence theory for them was established. The Broyden-like method was proposed and its global and local superlinear convergence conditions were explored in [24]. Recently, Xia and Li firstly extended the modulus-based matrix splitting method to solve nonlinear complementarity problems (1.1) and also discussed the global convergence when A is positive definite or an H_+-matrix [25]. The authors of this paper proposed the accelerated modulus-based matrix splitting iteration method for solving problem (1.1) and established the corresponding convergence theory both for positive definite system matrix and H_+-matrix in [26]. Two-step modulus-based matrix splitting iteration methods were presented in [27], the convergence was only studied when A is positive definite. In this paper, we further study the two-step modulus-based matrix splitting iteration method and analyze the corresponding convergence conditions when A is an H_+-matrix in detail. Numerical experiments verify the convergence theory and illustrate the efficiency of the method.

The rest of this paper is organized as follows. The two-step modulus-based matrix splitting iteration method for solving nonlinear complementarity problems (1.1) is proposed in