Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.2017.0003

Vol. **10**, No. 4, pp. 760-774
November 2017

Evaluation Algorithm of PHT-Spline Surfaces

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Received 9 January 2017; Accepted (in revised version) 22 March 2017

Abstract. PHT-splines are a type of polynomial splines over hierarchical T-meshes which posses perfect local refinement property. This property makes PHT-splines useful in geometric modeling and iso-geometric analysis. Current implementation of PHT-splines stores the basis functions in Bézier forms, which saves some computational costs but consumes a lot of memories. In this paper, we propose a de Boor like algorithm to evaluate PHT-splines provided that only the information about the control coefficients and the hierarchical mesh structure is given. The basic idea is to represent a PHT-spline locally in a tensor product B-spline, and then apply the de-Boor algorithm to evaluate the PHT-spline at a certain parameter pair. We perform analysis about computational complexity and memory costs. The results show that our algorithm takes about the same order of computational costs while requires much less amount of memory compared with the Bézier representations. We give an example to illustrate the effectiveness of our algorithm.

AMS subject classifications: TP391

Key words: PHT-splines, hierarchical T-mesh, knot insert, evaluation algorithm.

1. Introduction

In recent years, there have been much interest in locally refinable splines in the community of geometric modeling and iso-geometric analysis. Various such type of splines have been proposed, such as hierarchical B-splines, T-splines, PHT-splines, AST-splines, THB-splines, RHT-splines and LR-splines, etc [1–13, 17]. Compared with tensor product B-splines, locally refinable splines support local editing and eliminate superfluous control points in surface modeling. Furthermore, they are attractive in adaptive finite element methods and iso-geometric analysis.

In this paper, we focus on evaluation of PHT-splines–polynomial splines over hierarchical T-meshes. PHT-splines were firstly introduced by two of the present authors for adaptive surface modeling such as surface fitting, model simplification, etc [5]. The basis

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functions of PHT-splines posses many nice properties like B-spline functions, such as linear independency, partition of unity, compact support, nonnegativity, etc. Most of all, the basis functions of PHT-splines have perfect local refinement property, which makes PHT-splines very useful in adaptive surface modeling and iso-geometric analysis. PHT-splines have been successfully applied in solving partial differential equations [14], iso-geometric analysis [15–19], surface stitching [6] and implicit surface reconstructions [20]. Later on, PHT-splines were generalized over general T-meshes with arbitrary topology (GPT-splines, [7]). Theoretic study on the dimension of spline spaces over T-meshes can be found in [21–28].

For the current implementation of PHT-splines, the basis functions are computed and stored in Bézier forms beforehand [5]. The advantage is that computation with such representation is relatively efficient. Yet, the memory consumption is very large, especially for models with fine details and 3D implicit models [5,20]. To overcome such problem, in this paper we extend the de Boor algorithm for tensor-product B-splines to PHT-splines, where only the control coefficients and the hierarchical mesh structure are given. We perform analysis about computational complexity and memory consumption, and the results show that our algorithm takes about the same order of computational costs while requires much less amount of memory compared with the Bézier representations.

The paper is organized as follows. In Section 2, we review some preliminary knowledge about PHT-splines and the detailed basis construction process. Section 3 provides algorithm outline and Section 4 presents detailed algorithm. Section 5 gives an example to illustrate the algorithm. Section 6 analyzes the algorithm complexity together with a comparison with Bézier representations. We conclude the paper in Section 7.

2. PHT-splines

In this section, we recall some preliminary knowledge about PHT-splines and present detailed construction of the basis functions of PHT-splines.

2.1. Hierarchical T-mesh

A T-mesh is a rectangular grid that allows T-junctions on a 2D plane. In this paper, we adopt the same definitions of vertices, edges and cells as as in [5]. A *T-junction* is a vertex that lies in the interior of another edge. A *crossing-vertex* is the intersection of two interior edges. A *hierarchical T-mesh* is a special type of T-mesh that has a natural level structure. It is defined recursively. Generally, we start from a tensor-product mesh $\mathscr{T}_0$, which can be non-uniform and the elements (vertices, edges and cells) of which are called the level 0 elements. From level k to level $k+1$, some cells at level k are subdivided, with each cell is subdivided into 2×2 subcells, where the new vertices, the new edges and the new cells have level $k+1$, and the resulting T-mesh is called the level $k+1$ T-mesh, denoted as $\mathscr{T}_{k+1}$. Fig. 1 illustrates the process of generating a hierarchical T-mesh.