Numer. Math. Theor. Meth. Appl. Vol. **10**, No. 1, pp. 186-204
doi: 10.4208/nmtma.2017.m1613 February 2017

Total Variation Based Parameter-Free Model for Impulse Noise Removal

Federica Sciacchitano*, Yiqiu Dong and Martin S. Andersen

Department of Applied Mathematics and Computer Science, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark.

Received 14 March 2016; Accepted 13 June 2016

Abstract. We propose a new two-phase method for reconstruction of blurred images corrupted by impulse noise. In the first phase, we use a noise detector to identify the pixels that are contaminated by noise, and then, in the second phase, we reconstruct the noisy pixels by solving an equality constrained total variation minimization problem that preserves the exact values of the noise-free pixels. For images that are only corrupted by impulse noise (i.e., not blurred) we apply the semismooth Newton's method to a reduced problem, and if the images are also blurred, we solve the equality constrained reconstruction problem using a first-order primal-dual algorithm. The proposed model improves the computational efficiency (in the denoising case) and has the advantage of being regularization parameter-free. Our numerical results suggest that the method is competitive in terms of its restoration capabilities with respect to the other two-phase methods.

AMS subject classifications: 68U10, 94A08, 49J40, 52A41, 65K10, 90C47, 49M15

Key words: Image deblurring, image denoising, impulse noise, noise detector, primal-dual first-order algorithm, semismooth Newton method, total variation regularization.

1. Introduction

During the image acquisition and transmission, observed images are inevitably degraded by blur and noise. In the literature, many kinds of noise have been widely considered, Gaussian noise [14, 20, 36], impulse noise [7, 11, 28, 29, 31], multiplicative noise [3, 19, 35], Poisson noise [21, 26, 37] or mixed noise [8, 27, 38]. In this paper, we focus on blurred image with impulse noise, which is a common type of image degradation due to, e.g., malfunctioning pixel elements in the camera sensors, errors in analog-to-digital conversion, faulty memory locations in hardware, or transmission errors [5]. A characteristic property of impulse noise is that a certain number of pixels

*Corresponding author. *Email addresses:* feds@dtu.dk (F. Sciacchitano), yido@dtu.dk (Y. Dong), mskan@dtu.dk (M. S. Andersen)

are uncorrupted and the noise-corrupted pixels contain no information about the true pixel value.

Over the years, many nonlinear digital filters methods have been proposed, see [2]. The most common filters used to remove impulse noise are the median-type filters: median filter [34], weighted median filter [6], adaptive median filter [24], multistate median filter [15], center weighted median filter [25] and adaptive center-weighted median filter [16]. Although these filters are efficient and easy to implement, they cannot achieve good results in general, in particular they are not able to restore a blurred image and they do not preserve the image edges well.

In order to preserve the edges, in 2004, Nikolova [31] proposed a variational model which combines an ℓ^1-data fidelity term with total variation (TV), which has been in shown in [30, 31] to work better than the classical ℓ^2-term, [36].

Later, other approaches based on the ℓ^1-TV have been proposed to handle the deblurring problem and the non-differentiability of the ℓ^1-norm, for instance: Bar *et al.* [4] introduce a model using the Mumford-Shah regularizer and the ℓ^1-data fidelity term; Yang *et al.* [39] suggested an efficient algorithm to solve the ℓ^1-TV model; Dong *et al.* [17] solved the ℓ^1-TV model using a primal-dual approach.

However, since the ℓ^1-TV minimization method negatively affects the noisy-free pixels, in 2005, Chan, Ho, and Nikolova [12] proposed the so-called two-phase method. The basic idea behind this method, which we will refer to as the CHN method, is to separate noise detection and image reconstruction. In the first phase, the method uses a noise detector to identify which pixels are corrupted, and in the second phase, it reconstructs only the noisy pixels based on an objective function with an ℓ^1-data fidelity term and with TV as a regularization term. The two-phase model has also been studied for other applications, for instance in [8], the authors apply the two-phase method to restore blurred images with impulse and Gaussian noise; in [23], a two-phase method is used for recovering images corrupted by multiplicative noise; in [7] and [11], a two-phase method is used to simultaneously deblur and denoise an image with impulse noise. Different from [12], in the second phase of [7] and [11] the authors reconstruct the image based on a modified ℓ^1-TV model where only noise-free pixels are kept in the ℓ^1-data fidelity term, due to no useful information contained in impulse noise. We will focus only on the method in [11] (the CDH method in short), since it outperforms the one in [7] and [12] with respect to both image restoration capability and computational efficiency.

While the CDH method has been shown to perform well on many test problems, the inclusion of noise-free pixels in the data-fidelity term is somewhat at odds with the assumption that their true values are known. If the pixels are indeed noise-free, then they can either be treated as constants or eliminated from the problem. In this work, we investigate such an approach and propose a modified two-phase method. In particular, as suggested in [11], in the first phase we distinguish noisy pixels from the noise-free pixels by the adaptive median (AM) filter [24] for detecting salt-and-pepper noise, and the adaptive center-weighted median (ACWM) filter [16] for random-valued impulse noise. The detector for salt-and-pepper noise is able to detect almost all noisy