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Transport Phenomena and Mixing Induced by Vortex Formation in Flow Around Airfoil Using Lagrangian Coherent Structures

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Abstract. In this study, mass transport between separation bubbles and the flow around a two-dimensional airfoil are numerically investigated using Lagrangian Coherent Structures (LCSs). Finite Time Lyapunov Exponent (FTLE) technique is used for the computation to identify invariant manifolds and LCSs. Moreover, the Characteristic Base Split (CBS) scheme combined with dual time stepping technique is applied to simulate such transient flow at low Reynolds number. We then investigate the evolution of vortex structures during the transport process with the aid of LCSs. To explore the vortex formation at the surface of the airfoil, the dynamics of separatrix is also taken into account which is formed by the combination of stable (unstable) manifold. The Lagrangian analysis gives a detailed understanding of vortex dynamics and separation bubbles which plays a significant role to explore the performance of the unsteady flow generated by an airfoil. Transport process and flow separation phenomena are studied extensively to analyze the flow pattern by the Lagrangian point of view.

AMS subject classifications: 76M10, 80M10, 76D17, 37L25
Key words: Transport phenomena, CBS method, vortex formation, Lagrangian coherent structures, stable (unstable) manifold.

1. Introduction

Vortex shedding is ubiquitous in many engineering applications and biological systems such as, fluid transport phenomena, flow control, insect fligt and fish swimming [1–4]. The flow around an airfoil has been the subject of intense research in the last few decades and various numerical, experimental and theoretical investigations can be found in the literature [5–10]. In these studies, Eulerian description of

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the flow pattern is analyzed, for example, vorticity and streamline. However, Eulerian flow description cannot reveal the dynamic properties of fluid structure. Therefore, researchers prefer to apply Lagrangian description of the fluid to study such non-linear dynamical systems.

The Lagrangian approach to the characterization of the transport and mixing properties of fluids in terms of coherent structures has attracted much attention in recent years. The study of non-linear dynamical systems and Lagrangian transport in fluid has become an important subject for deeper understanding of Lagrangian structures in different fluid flows. Lagrangian view which determined finite time material lines by establishing the Finite-Time Lyapunov Exponent (FTLE) in a velocity field is described by Haller [11]. Exponential divergence of nearby particle trajectories is determined by FTLE in the flow. As, the particle tracing is linear in the kinematic fluid, the FTLE may be explained in either back or forth time direction. FTLE measures the stretching in forwarding time. In the contrary, FTLE determine the contraction of the fluid in backward time. In [12] a bunch of intersecting a repelling and an attracting transport barrier enclosing a circulating fluid was presented to be an excellent identifier of a vortex. The deformation of these FTLE enclosures over time is instrumental in measuring coherent structures [13] and fluid entrainment mechanisms in a time-dependent flow [14]. To study various physics and engineering problems, these coherent strutures were visualized by using various tools [15, 16]. However, recently mathematical point of view is established to define and identify LCSs [17–19].

Dynamical systems tools and concepts: LCSs, stable and unstable manifolds, lobe structure and evolution have been studied in [20]. An analytical approach has been introduced to describe the significance of invariant manifolds and the manner by which these structures control transport. These dynamical systems tools and concepts are extremely useful and insightful in studies involving fluid transport. Unsteady boundary layer equation in the Lagrangian frame is studied by Van Dommelen [21], and he described the flow separation criteria through Lagrangian description. Duan and Wiggins [22] explained the mass transport and vortex shedding near the wake of a circular cylinder using lobe dynamics and manifolds theorem. However, non-linear dynamics can be approached in infinite time flow by traditional manifold. The concepts of finite time manifolds and LCSs are studied by Haller [23] and define them to analyze the boundaries of the vortex in finite time flow. Researchers are convenient to apply the Lagrangian method in studying the unsteady separation and fluid transport. A wide range of flow phenomena, such as biological phenomena [24], turbulent flows [25], vortex shedding [26], geophysical flows [27] have been studied using LCSs. Guo and Lin [28] analyzed an unsteady, viscous and incompressible flow of a micropolar fluid in a uniformly porous channel, whereas Cheng et al. [29] applied a second-order cell-centered Lagrangian method to study a two-dimensional elastic-plastic flow.

We are primarily interested in deep understanding of evolution of flow structure around an airfoil. In Section 2, we provide a brief review of Lagrangian Coherent Structures to understand Lagrangian dynamics of the flow phenomena like vortex and separation bubble. In Sections 3, we introduced a numerical method to simulate the