

Numerical Integration Based on Bivariate Quartic Quasi-Interpolation Operators

Renhong Wang

(Institute of Mathematical Sciences, Dalian University of Technology, Dalian 116024, China

E-mail: renhong@dlut.edu.cn)

Xiaolei Zhang*

(Institute of Mathematical Sciences, Dalian University of Technology, Dalian 116024, China;

and Department of Applied Mathematics, Changchun University of Science and Technology,
Changchun 130022, China

E-mail: zhangxl0411@yahoo.com.cn)

Received October 17, 2006; Accepted (in revised version) November 8, 2006

Abstract

In this paper, we propose a method to deal with numerical integral by using two kinds of C^2 quasi-interpolation operators on the bivariate spline space, and also discuss the convergence properties and error estimates. Moreover, the proposed method is applied to the numerical evaluation of 2-D singular integrals. Numerical experiments will be carried out and the results will be compared with some previously published results.

Keywords: $S_4^2(\Delta_{mn}^{(2)})$; quasi-interpolation operators; singular integrals.

Mathematics subject classification: 65D32, 65D30, 41A15

1. Introduction

It is well known that multivariate splines play an important role in both theories and applications in science and engineering (see [10] and references therein). Numerical evaluation of the two-dimensional singular integrals based on quasi-interpolation operators on bivariate spline spaces have been studied in [2, 3, 8].

In [3], the numerical integration is investigated based on the quasi-interpolation operator on $S_2^1(\Delta_{mn}^{(2)})$. The operator in [3] only preserves the polynomial of degree 1, and this motivates us to adopt quasi-interpolation operators which possess higher polynomial reproduction and thus obtain better approximation behavior.

The purpose of this paper is to deal with the problem of two-dimensional numerical integration based on the bivariate C^2 quasi-interpolating splines. Let $D = [0, 1] \otimes [0, 1]$ and the type-2 triangulation (four-directional mesh) $\Delta_{mn}^{(2)}$ be given by the following grid lines:

$$mx - i = 0, \quad ny - i = 0, \quad ny - mx - i = 0, \quad ny + mx - i = 0,$$

where i are all integers, m, n are given integers.

*Corresponding author.

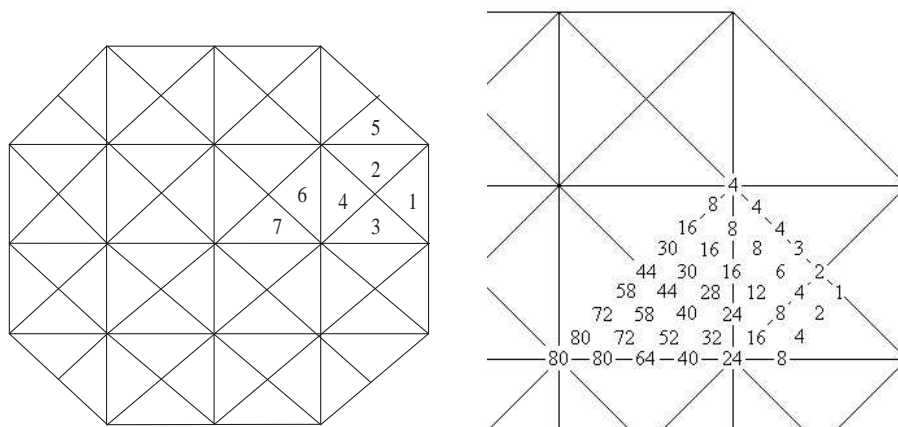


Fig. 2.1. The support and Bézier coefficients of $B(x, y)$.

The organization of the paper is as follows. In Section 2 we recall some properties of the bivariate spline space $S_4^2(\Delta_{mn}^{(2)})$ and two kinds of quasi-interpolation operators. Some of their polynomial preserving and approximation properties are given. In Section 3 the numerical integration formula based on the quasi-interpolation operators is obtained and their convergence properties and error estimates are also given. Finally, the numerical evaluation of singular integrals defined in the Hadamard finite part sense based on the proposed integration formulas are given in Section 4.

2. C^2 quasi-interpolation operators on quartic spline space

In [1, 10], the locally supported splines in $S_4^2(\Delta_{mn}^{(2)})$ are constructed. Because the basis of $S_4^2(\Delta_{mn}^{(2)})$ are comprised of three C^2 quartic splines, it is not convenient to use them immediately. A new spline $B(x, y)$ is constructed by using a linear combination of three kinds of B-splines whose support and Bézier representations as shown in Fig. 2.1, where the center of the support is at $(0, 0)$. Here, the Bézier coefficients (B-net) which are not shown are either zero or can be obtained by symmetry. Moreover, to avoid fraction the coefficients are multiplied by 192 [7, 11].

It is proved that all the locally supported splines can only span a proper subspace of $S_4^2(\Delta_{mn}^{(2)})$. Indeed, it spans a proper subspace of $S_4^{2,3}(\Delta_{mn}^{(2)})$ [11]. Since it possesses good symmetry, the computation cost will be reduced greatly when applied to numerical integrations.

Denote

$$B_{ij}(x, y) = B(mx - i, ny - j), \quad i = -1, 0, \dots, m + 1, \quad j = -1, 0, \dots, n + 1.$$

It is clear that $(x_i, y_j) = (\frac{i}{m}, \frac{j}{n})$ is the center of the support Q_{ij} of $B_{ij}(x, y)$. Now, we consider the following bivariate variation diminishing spline operators:

$$V_{mn}^{(j)} : C(\Omega) \rightarrow S_4^{2,3}(\Delta_{mn}^{(2)})$$