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# Block Based Bivariate Blending Rational Interpolation via Symmetric Branched Continued Fractions†

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**Abstract.** This paper constructs a new kind of block based bivariate blending rational interpolation via symmetric branched continued fractions. The construction process may be outlined as follows. The first step is to divide the original set of support points into some subsets (blocks). Then construct each block by using symmetric branched continued fraction. Finally assemble these blocks by Newton's method to shape the whole interpolation scheme. Our new method offers many flexible bivariate blending rational interpolation schemes which include the classical bivariate Newton's polynomial interpolation and symmetric branched continued fraction interpolation as its special cases. The block based bivariate blending rational interpolation is in fact a kind of tradeoff between the purely linear interpolation and the purely nonlinear interpolation. Finally, numerical examples are given to show the effectiveness of the proposed method.

**Key words**: Interpolation; block based bivariate partial divided differences; symmetric branched continued fractions; blending method.

**AMS subject classifications:** 41A20, 65D05

## 1 Introduction

Bivariate Newton's polynomial interpolation may be the most commonly used bivariate interpolation. It uses the bivariate partial divided differences which can be calculated recursively and produce useful intermediate results. On the other hand, the most powerful bivariate interpolation is the one using bivariate rational functions. The main advantage of the rational functions over polynomials is their ability to model functions with nonlinear characters (such as

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poles or other singularity) and their fast convergence properties. Given a set of two dimensional points $\Pi_{mn} = \{(x_i, y_j) \mid i = 0, 1, \ldots, m; j = 0, 1, \ldots, n\}$, and suppose that $f(x, y)$ is defined on $D \supset \Pi_{mn}$. Then one has two basic approaches for interpolating $f(x, y)$ on $\Pi_{mn}$. One is the bivariate Newton's interpolating polynomial ([5])

$$P(x,y) = \sum_{i=0}^{m}\sum_{j=0}^{n} f[x_0,\ldots,x_i;y_0,\ldots,y_j]\prod_{h=0}^{i-1}(x-x_h)\prod_{k=0}^{j-1}(y-y_k),$$

where the empty products are defined to take the value 1, and

$$\begin{aligned}
&f[x_0;y_0] = f(x_0,y_0),\\
&f[x_0,\ldots,x_i;y_0] = \frac{f[x_1,\ldots,x_i;y_0]-f[x_0,\ldots,x_{i-1};y_0]}{x_i-x_0},\\
&f[x_0,\ldots,x_i;y_0,\ldots,y_j] = \frac{f[x_0,\ldots,x_i;y_1,\ldots,y_j]-f[x_0,\ldots,x_i;y_0,\ldots,y_{j-1}]}{y_j-y_0}.
\end{aligned}$$

The other one is the interpolating symmetric branched continued fraction ([2–4, 7])

$$\begin{aligned}
R(x,y) \;=\;& \varphi_{00} + \sum_{k=1}^{m}\frac{x-x_{k-1}|}{|\varphi_{k0}} + \sum_{k=1}^{n}\frac{y-y_{k-1}|}{|\varphi_{0k}}\\
&+\sum_{l=1}^{m}\frac{(x-x_{l-1})(y-y_{l-1})|}{|\varphi_{ll}+\sum_{k=l+1}^{m}\frac{x-x_{k-1}|}{|\varphi_{kl}}+\sum_{k=l+1}^{n}\frac{y-y_{k-1}|}{|\varphi_{lk}}},\ (m\le n)
\end{aligned}$$

or

$$\begin{aligned}
R(x,y) \;=\;& \varphi_{00} + \sum_{k=1}^{m}\frac{x-x_{k-1}|}{|\varphi_{k0}} + \sum_{k=1}^{n}\frac{y-y_{k-1}|}{|\varphi_{0k}}\\
&+\sum_{l=1}^{n}\frac{(x-x_{l-1})(y-y_{l-1})|}{|\varphi_{ll}+\sum_{k=l+1}^{m}\frac{x-x_{k-1}|}{|\varphi_{kl}}+\sum_{k=l+1}^{n}\frac{y-y_{k-1}|}{|\varphi_{lk}}},\ (n\le m)
\end{aligned}$$

where $\varphi_{ij} = \varphi[x_0,\ldots,x_i;y_0,\ldots,y_j]$, and

$$\begin{aligned}
&\varphi[x_0;y_0] = f(x_0,y_0),\\
&\varphi[x_0,\ldots,x_k;y_0] = \frac{x_k-x_{k-1}}{\varphi[x_0,\ldots,x_{k-2},x_k;y_0]-\varphi[x_0,\ldots,x_{k-1};y_0]},\\
&\varphi[x_0;y_0,\ldots,y_k] = \frac{y_k-y_{k-1}}{\varphi[x_0;y_0,\ldots,y_{k-2},y_k]-\varphi[x_0;y_0,\ldots,y_{k-1}]},\\
&\varphi[x_0,\ldots,x_j;y_0,\ldots,y_j] = (x_j-x_{j-1})(y_j-y_{j-1})(\varphi[x_0,\ldots,x_{j-2},x_j;y_0,\ldots,y_{j-2},y_j]\\
&\qquad-\varphi[x_0,\ldots,x_{j-1};y_0,\ldots,y_{j-2},y_j]-\varphi[x_0,\ldots,x_{j-2},x_j;y_0,\ldots,y_{j-1}]\\
&\qquad+\varphi[x_0,\ldots,x_{j-1};y_0,\ldots,y_{j-1}])^{-1},
\end{aligned}$$

and for $k > j$

$$\begin{aligned}
\varphi[x_0,\ldots,x_k;y_0,\ldots,y_j] &= \frac{x_k-x_{k-1}}{\varphi[x_0,\ldots,x_{k-2},x_k;y_0,\ldots,y_j]-\varphi[x_0,\ldots,x_{k-1};y_0,\ldots,y_j]},\\
\varphi[x_0,\ldots,x_j;y_0,\ldots,y_k] &= \frac{y_k-y_{k-1}}{\varphi[x_0,\ldots,x_j;y_0,\ldots,y_{k-2},y_k]-\varphi[x_0,\ldots,x_j;y_0,\ldots,y_{k-1}]}.
\end{aligned}$$