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On Choices of Stress Modes for Lower Order Quadrilateral Reissner-Mindlin Plate Elements†

Guanghui Hu* and Xiaoping Xie

School of Mathematics, Sichuan University, Chengdu 610064, China.

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Abstract. A kind of stabilized mixed/hybrid scheme for Reissner-Mindlin plates is proposed with conforming isoparametric bilinear interpolations of deflection/rotations. The choice of shear stress modes is discussed. It is shown by numerical experiments that fulfilling an energy orthogonal condition for stress approximations is crucial to avoiding "shear locking".

Key words: Mixed/hybrid finite element; Reissner-Mindlin plate; locking-free.

AMS subject classifications: 65N12, 65N15, 65N30

1 Introduction

In recent years, a lot of work for the Mindlin-Reissner (R-M) plate model has been done in the engineering and mathematical literatures (see [1-5, 7-15, 17] and references therein). As one knows, one of the most important problems is how to avoid the locking phenomena in the thin plate case.

Among the existing approaches, a stabilizing technique has often been used [10, 11, 14, 17] to avoid the "shear locking". In this paper, we will discuss the influence of stress choices to the "locking" for stabilized lower order quadrilateral R-M plate elements with conforming isoparametric bilinear interpolations for approximations of the deflection/rotations.

Let $\Omega \subset R^2$ be the midsurface of the plate. The variational problem for the Mindlin-Reissner plate bending model with clamped boundary reads as: find the deflection $\omega \in H_0^1(\Omega)$ and the rotation vector $\beta \in [H_0^1(\Omega)]^2$, such that

$$a(\beta,\zeta) + \frac{\lambda}{t^2}(\bigtriangledown\omega - \beta, \bigtriangledown v - \zeta) = (f, v) \qquad \forall (v,\zeta) \in [H_0^1(\Omega)]^3, \tag{1}$$

where t is the thickness of the plate, $\lambda = \dfrac{5E}{12(1+\nu)}$, with E the Young's modulus and ν the Poisson ratio, f is the transverse load, $a(\beta,\zeta) := \displaystyle\int_\Omega \epsilon(\beta) : D_b \epsilon(\zeta) \mathrm{d}\Omega$, with $\epsilon(\beta) = \frac{1}{2}[\nabla\beta + (\nabla\beta)^T]$

*Correspondence to: Guanghui Hu, School of Mathematics, Sichuan University, Chengdu 610064, China. Email: hghdd@sina.com
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the bending strain tensor and

$$\mathbf{D_b} = \frac{E}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

the elasticity matrix.

By introducing the scaled shear stress $\gamma = \lambda(\bigtriangledown\omega - \beta)/t^2$ as an independent unknown, the mixed/hybrid form of (1) is: find $(\omega, \beta; \gamma) \in H_0^1(\Omega) \times [H_0^1(\Omega)]^2 \times [L_2(\Omega)]^2$ such that

$$\begin{aligned} a(\beta,\zeta) + (\bigtriangledown v - \zeta, \gamma) &= (f, v), \quad \forall (v,\zeta) \in H_0^1(\Omega) \times [H_0^1(\Omega)]^2, & (2) \\ \frac{t^2}{\lambda}(\gamma,\tau) - (\bigtriangledown\omega - \beta, \tau) &= 0, \qquad \forall \tau \in [L_2(\Omega)]^2. & (3) \end{aligned}$$

The potential energy functional for (1) and the mixed/hybrid energy functional for (2)-(3) are respectively of the following forms:

$$\begin{aligned} \Pi_p(v,\zeta) &= \frac{1}{2}\left(a(\zeta,\zeta) + \frac{\lambda}{t^2}(\bigtriangledown v - \zeta, \bigtriangledown v - \zeta)\right) - (f, v), & (4) \\ \Pi_{HR}(v,\zeta;\tau) &= \frac{1}{2}\left(a(\zeta,\zeta) - \frac{t^2}{\lambda}(\tau,\tau) + 2(\tau, \bigtriangledown v - \zeta)\right) - (f, v) \\ &= \Pi_p(v,\zeta) - \frac{t^2}{2\lambda}(\tau - \lambda t^{-2}(\bigtriangledown v - \zeta), \tau - \lambda t^{-2}(\bigtriangledown v - \zeta)). & (5) \end{aligned}$$

Thus, we have:

$$\begin{aligned} \Pi_{HR}(\omega,\beta;\gamma) &= \inf_{(v,\zeta)} \sup_{\tau} \Pi_{HR}(v,\zeta;\tau) \\ &= \inf_{(v,\zeta)} \left(\Pi_p(v,\zeta) - \inf_{\tau} \frac{t^2}{2\lambda}(\tau - \lambda t^{-2}(\bigtriangledown v - \zeta), \tau - \lambda t^{-2}(\bigtriangledown v - \zeta))\right), \end{aligned}$$

which means, in discretized cases, that the energy of the mixed/hybrid model is always no greater than that of the potential energy model; that a much bigger stress approximation subspace can lead to a much bigger (thus bad) energy approximation of the mixed/hybrid finite element model.

2 The finite element method and stress choices

Let C_h be the finite element partitioning of $\overline{\Omega}$ into convex quadrilaterals and define the finite element subspaces for the deflection and rotation vector as

$$\begin{aligned} W_h &:= \{v \in H_0^1(\Omega) : v|_K \in \mathrm{span}\{1, \xi, \eta, \xi\eta\}, \quad \forall K \in C_h\}, \\ V_h &:= \{\zeta \in [H_0^1(\Omega)]^2 : \zeta|_K \in [\mathrm{span}\{1, \xi, \eta, \xi\eta\}]^2, \quad \forall K \in C_h\}, \end{aligned}$$

where ξ, η are the isoparametric coordinates, and the isoparametric mapping $F_K : \widehat{K} = [-1,1]^2 \to K$ is given by

$$\left\{ \begin{array}{c} x \\ y \end{array} \right\} = F_K(\xi,\eta) = \frac{1}{4}\sum_{i=1}^{4}(1+\xi_i\xi)(1+\eta_i\eta) \left\{ \begin{array}{c} x_i \\ y_i \end{array} \right\} = \left\{ \begin{array}{c} a_0 + a_1\xi + a_2\xi\eta + a_3\eta \\ b_0 + b_1\xi + b_2\xi\eta + b_3\eta \end{array} \right\}.$$