English Series | NUMERICAL MATHEMATICS A Journal of Chinese Universities | Vol.14, No.2 May 2005

SOLUTION OF BACKWARD HEAT PROBLEM BY MOROZOV DISCREPANCY PRINCIPLE AND CONDITIONAL STABILITY*

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Abstract *Consider a 1-D backward heat conduction problem with Robin boundary condition. We recover $u(x,0)$ and $u(x,t_0)$ for $t_0 \in (0,T)$ from the measured data $u(x,T)$ respectively. The first problem is solved by the Morozov discrepancy principle for which a 3-order iteration procedure is applied to determine the regularizing parameter. For the second one, we combine the conditional stability with the Tikhonov regularization together to construct the regularizing solution for which the convergence rate is also established. Numerical results are given to show the validity of our inversion method*

Key words *Backward heat problem, regularization, conditional stability, convergence, numerics.*

AMS(2000)subject classifications 35R30, 35J05

1 Introduction

Consider the following 1-D heat conduction problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}(k(x)\dfrac{\partial u}{\partial x}), & (x,t) \in (a,b) \times (0,T), \\ u_x(a,t) - h(a)u(a,t) = 0, & t \in (0,T), \\ u_x(b,t) + h(b)u(b,t) = 0, & t \in (0,T), \\ u(x,0) = g(x), & x \in (a,b) \end{cases} \tag{1.1}$$

with positive $k(x)$ and $h(a), h(b) \geq 0$, which describes the heat conduction within a stick without heat sink or heat source. The classical direct problem is to find $u(x,t)$ for $t > 0$ from given initial temperature $g(x)$. This direct problem can be solved by difference schemes such as Crank-

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Received: Dec. 28, 2004.

Nicolson scheme or implicit Euler scheme.

However, in many practical application areas such as archeology, it is also necessary to find the temperature $u(x,t)$ for $t \in [0,T)$ from the known final value $u(x,T)$. This is the so-called backward heat problem which is well-known to be ill-posed, namely, the solution does not depend continuously on the input data $u(x,T)$. In fact, the rapid decay of temperature with respect to time t results in the quick disappearance of characteristic of $g(x)$. Therefore the numerical recovery of initial temperature from measured data at time $T > 0$ is very difficult due to the input data error and computational error.

There have been much research works on the backward heat problem, for example, see [1,2,3,4,11] and the references therein. In [1], R.Chapko used the regularized Newton method to discuss a 2-D heat conduction problem of recovering the medium boundary, while H.Han considered the 1-D backward heat problem by the minimum energy technique and the boundary element method([2]). In 1999, Muniz([3]) tested three numerical methods for solving 1-D backward heat problem with Dirichlet boundary condition. The first method converts the problem to an integral equation of the first kind and solves it directly. Of course, the results are not satisfactory. In his second method, the backward implicit Euler scheme is used to get satisfactory numerical results for exact input data. Finally he used the Tikhonov regularization method to this problem and obtained very good numerical results. In [4,5,10], the Tikhonov regularization is used to consider the 2-D backward heat problems with Dirichlet boundary condition. On the choice of regularization parameter for the Tikhonov method, except for the well-known strategy, some new techniques have been developed([6],[7]). For example, the conditional stability is applied to give a choice strategy of regularization parameter and analyze the convergence rate.

In this paper, we consider the 1-D backward heat conduction problem described by (1.1). Our problem is to determine $u(x,t_0)$ for $t_0 \in [0,T)$ from the noisy data of $u(x,T)$. We apply the Morozov discrepancy principle to recover $u(x,0)$. For $t_0 \in (0,T)$, the conditional stability result is applied to construct the regularizing solution of $u(x,t_0)$ for which the convergence rate is also given. It is interesting that the convergence rate for $t_0 \in (0,T)$ is invalid for $t_0 = 0$.

This paper is organized as follows. Firstly we convert this problem into an integral equation and show the ill-posedness of the problem by logarithmic convex method in section 2. Then we apply the Morozov discrepancy principle to recover $u(x,0)$ and construct an iteration procedure with 3-order convergence rate for the regularizing parameter in section 3. In section 4, we construct the regularizing solution of $u(x,t_0)$ for $t_0 > 0$ by the conditional stability result. Finally we give some numerical results in section 5 to show the validity of our inversion method.