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PPA BASED PREDICTION-CORRECTION METHODS FOR MONOTONE VARIATIONAL INEQUALITIES*

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Abstract *In this paper we study the proximal point algorithm (PPA) based prediction-correction (PC) methods for monotone variational inequalities. Each iteration of these methods consists of a prediction and a correction. The predictors are produced by inexact PPA steps. The new iterates are then updated by a correction using the PPA formula. We present two profit functions which serve two purposes: First we show that the profit functions are tight lower bounds of the improvements obtained in each iteration. Based on this conclusion we obtain the convergence inexactness restrictions for the prediction step. Second we show that the profit functions are quadratically dependent upon the step lengths, thus the optimal step lengths are obtained in the correction step. In the last part of the paper we compare the strengths of different methods based on their inexactness restrictions.*

Key words *Monotone variational inequality, proximal point algorithm, prediction-correction method.*

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1 Introduction

Let Ω be a nonempty closed convex subset of R^n and F be a continuous monotone mapping from R^n into itself. The variational inequality problem is to determine a vector $u^* \in \Omega$ such that

$$\mathrm{VI}(\Omega, F) \qquad (u-u^*)^T F(u^*) \ge 0, \qquad \forall u \in \Omega. \tag{1.1}$$

$\mathrm{VI}(\Omega, F)$ problems include nonlinear complementarity problems (when $\Omega = R^n_+$) and systems of

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nonlinear equations (when $\Omega = R^n$), and thus have many important applications [7,8,9].

A classical method for solving variational inequality is the proximal point algorithm (abbreviated as PPA) [17,18]. Given $u^k \in \Omega$ and $\beta_k > 0$, the new iterate u^{k+1} of PPA is obtained by solving the following variational inequality:

$$\text{(PPA)} \qquad u \in \Omega, \qquad (u' - u)^T F_k(u) \geq 0, \qquad \forall u' \in \Omega, \tag{1.2}$$

where

$$F_k(u) = (u - u^k) + \beta_k F(u). \tag{1.3}$$

An equivalent recursion form of PPA is

$$u^{k+1} = P_\Omega[u^{k+1} - F_k(u^{k+1})], \tag{1.4}$$

where P_Ω denotes the projection on Ω. The above projection equation can be written as

$$u^{k+1} = P_\Omega[u^k - \beta_k F(u^{k+1})]. \tag{1.5}$$

Since u^{k+1} occurs on both sides of equation (1.5), we call the method an *implicit* method [11].

The ideal form (1.5) of the method is often impractical since in many cases solving problem (1.2) exactly is either impossible or expensive. In 1976 Rockafellar set up the fundamental convergence analysis for the approximate proximal point algorithm (abbreviated as APPA) to a general maximal monotone operator [17]. Extensive developments on APPA followed, focusing on different fields such as convex programming, mini-max problems, and variational inequality problems. To mention a few, see [1, 3, 4, 5, 6, 16]. The major challenges of such methods include setting the restrictions of the approximation which are both easy to implement and tight for convergence, and accelerating the convergence.

In this paper, we study a particular group of methods which share the flavor of APPA. We call the methods proximal point algorithm based prediction-correction methods (abbreviated as PPA-PC methods). Given $u^k \in \Omega$ and $\beta_k > 0$, let v^k be an approximate solution of (1.2) in the sense that

$$v^k \approx P_\Omega[v^k - F_k(v^k)] \tag{1.6}$$

and define

$$\tilde{v}^k := P_\Omega[v^k - F_k(v^k)]. \tag{1.7}$$

The new iterate of these methods is given by either

$$\text{(PPA-PC1)} \qquad u^{k+1}(\alpha, v^k) = P_\Omega[u^k - \alpha\beta_k F(v^k)] \tag{1.8}$$

or

$$\text{(PPA-PC2)} \qquad u^{k+1}(\alpha, \tilde{v}^k) = P_\Omega[u^k - \alpha\beta_k F(\tilde{v}^k)]. \tag{1.9}$$

In such methods, v^k and $\tilde{v}^k$ can be viewed as predictors generated by inexactly solving the variational inequality (1.2). Ignoring the step length α, the new iterate u^{k+1} in (1.8) (resp. in (1.9)) can be viewed as the corrector obtained from equation (1.5) via substituting the u^{k+1} in the right hand side by the predictor v^k (resp. $\tilde{v}^k$). Therefore, we refer (1.8) and (1.9) as PPA based